Programmable Metasurfaces for Wireless Communications: A Loaded Thin Wire Model

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H2020-MSCA-ITN-2020 Meta Wireless

COST ACTION INTERACT

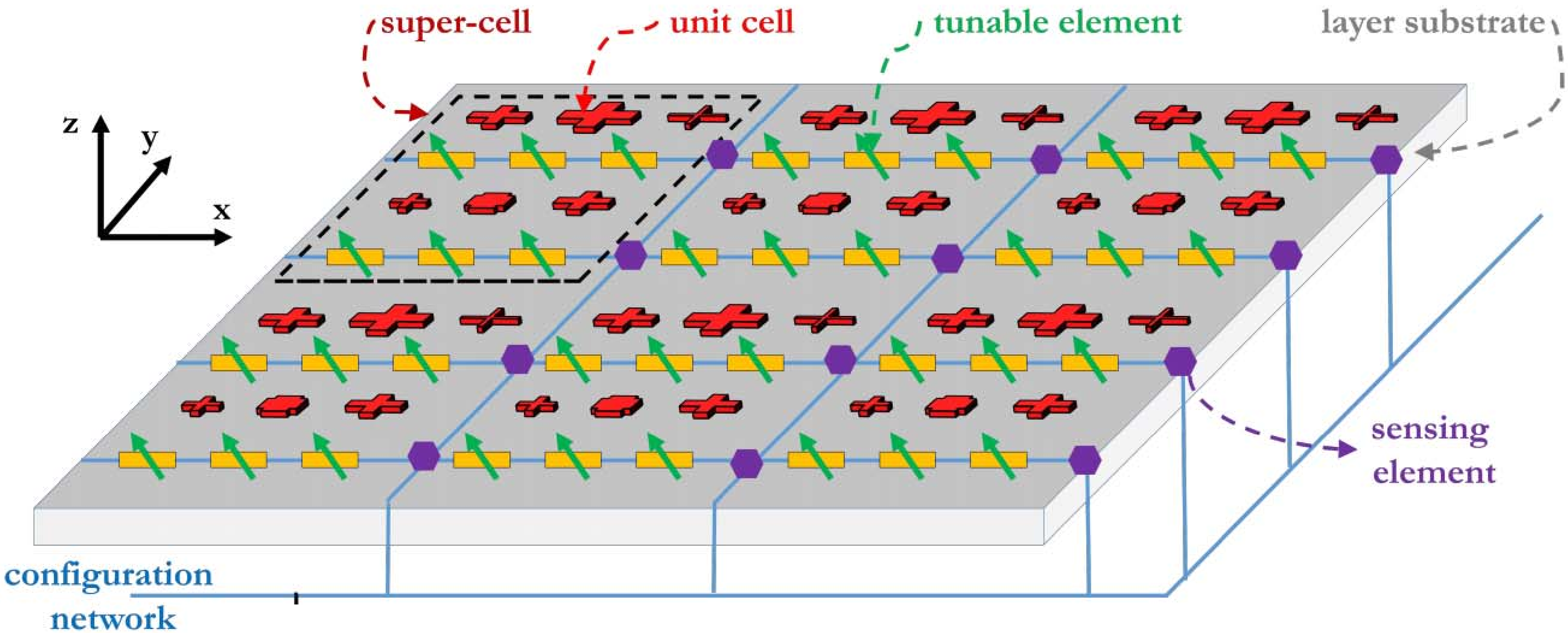
H2020-2018-2020, ICT ARIADNE

H2020-52-2020, ICT RISE-6G

6G Wireless Foundations Forum
Sophia Antipolis, France
July 10, 2023

What is a
Programmable Metasurface?

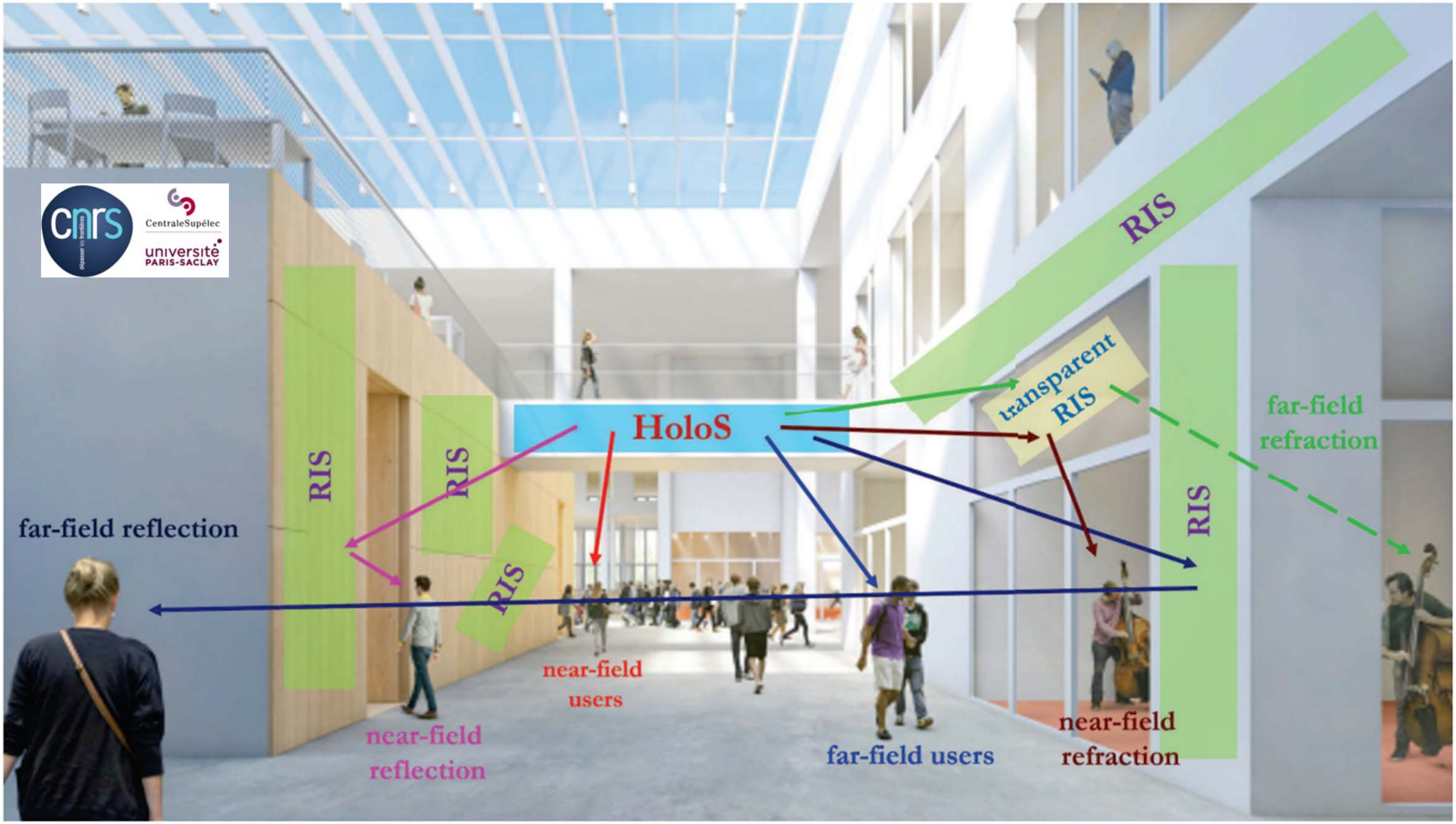
Programmable Metasurfaces



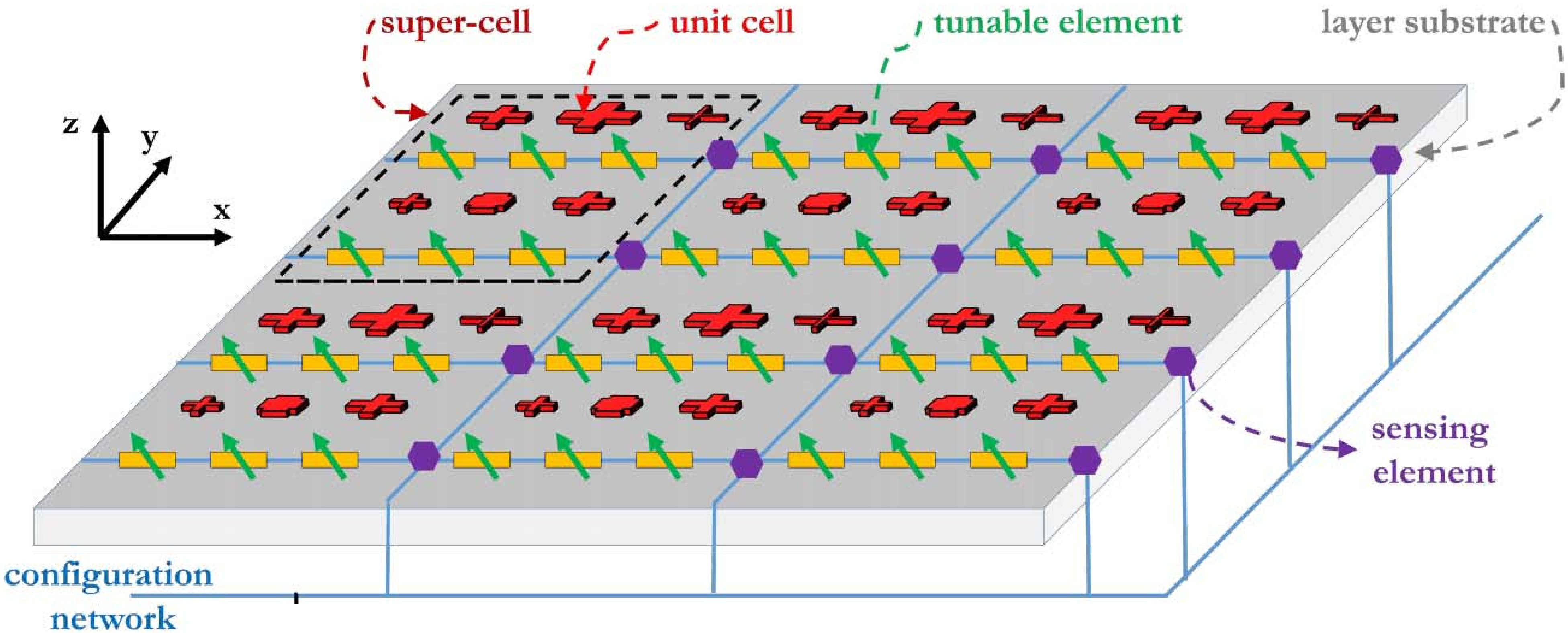
M. Di Renzo et al. Smart radio environments empowered by reconfigurable intelligent surfaces: How it works, state of research, and the road ahead. IEEE JSAC, Nov. 2020.

Towards Smart Radio Environments

Smart (Reconfigurable) Radio Environment



Programmable Metasurfaces



Main features (RIS):

- No power amplifiers
- No digital signal processing
- No radio frequency chains

Main features (HoloS):

- Power amplifiers
- Digital signal processing
- Radio frequency chains

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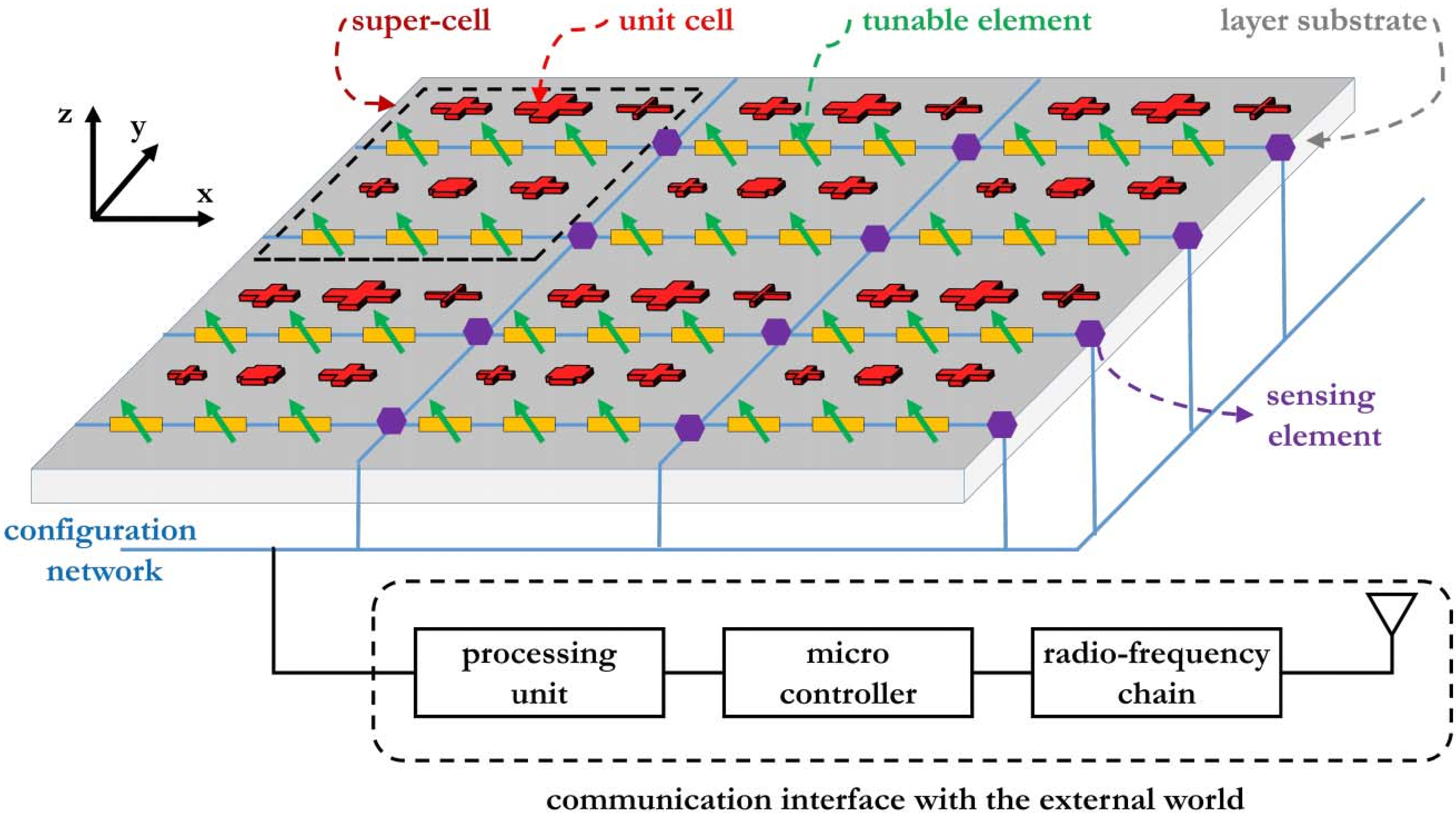
Programmable Metasurfaces (RIS)



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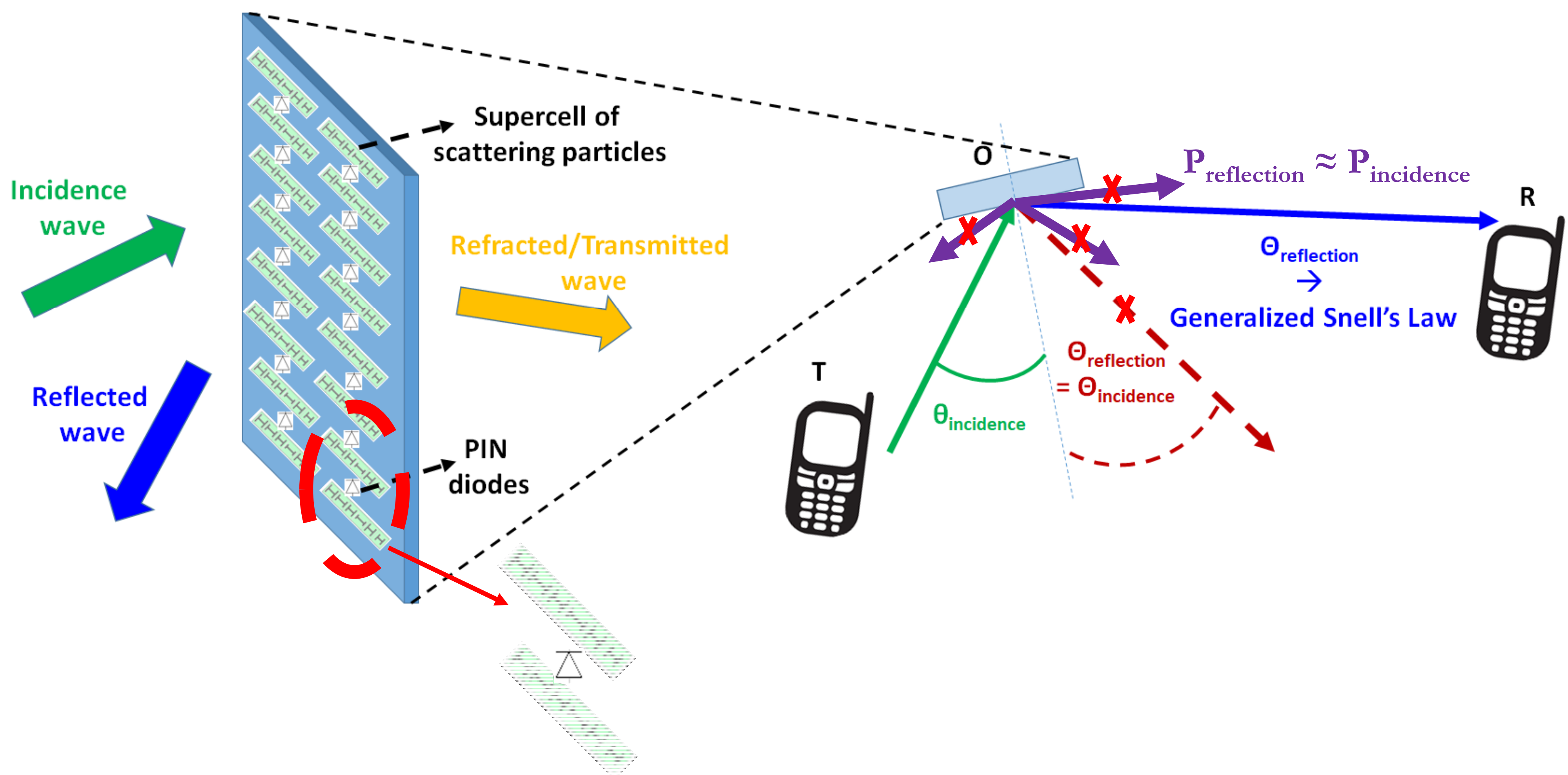
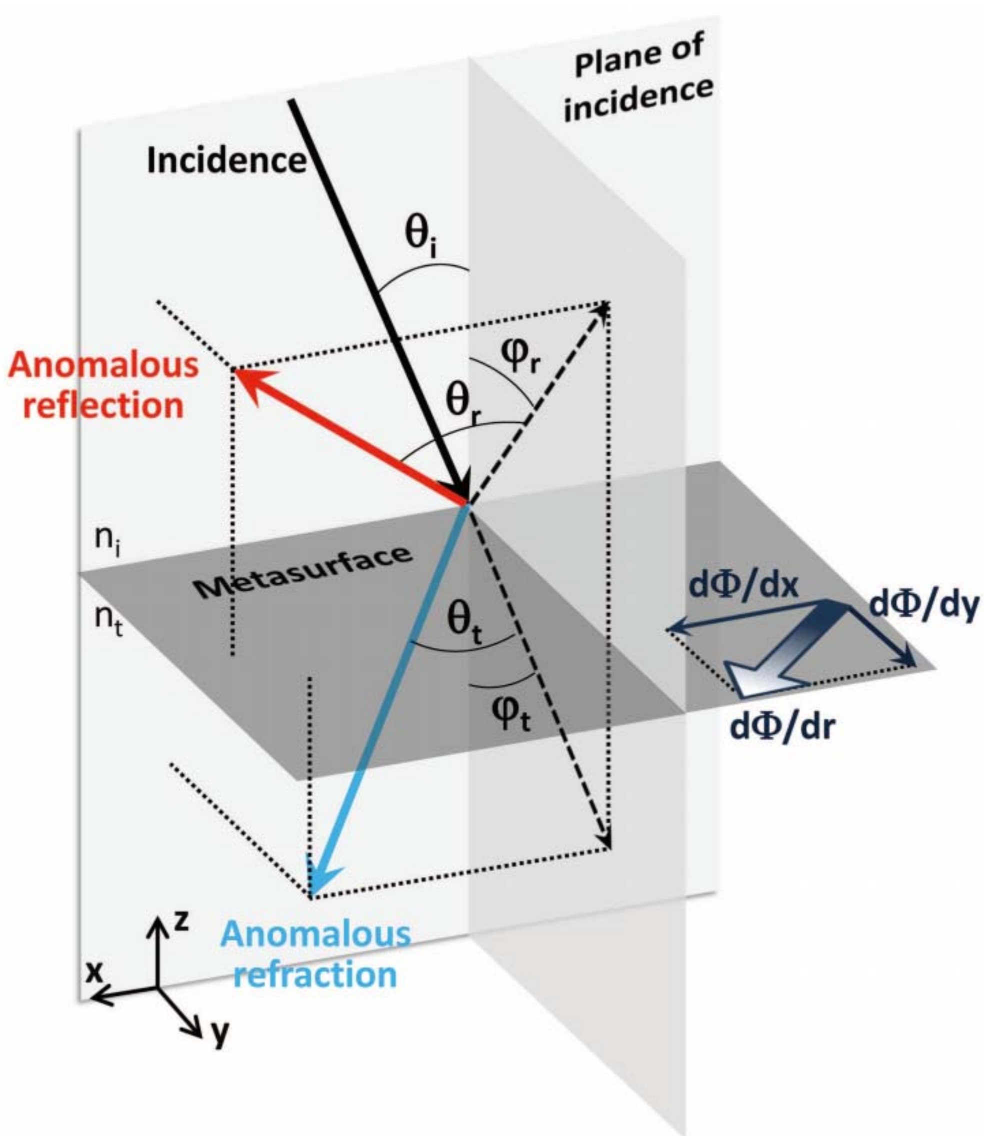
RIS Example:
Reflecting Metasurface

Reflecting Metasurface (RIS)



Generalized Snell's Law

Generalized Snell's Law



$$n_t \sin(\theta_t) - n_i \sin(\theta_i) = \frac{1}{k_0}\frac{d\Phi}{dx}$$

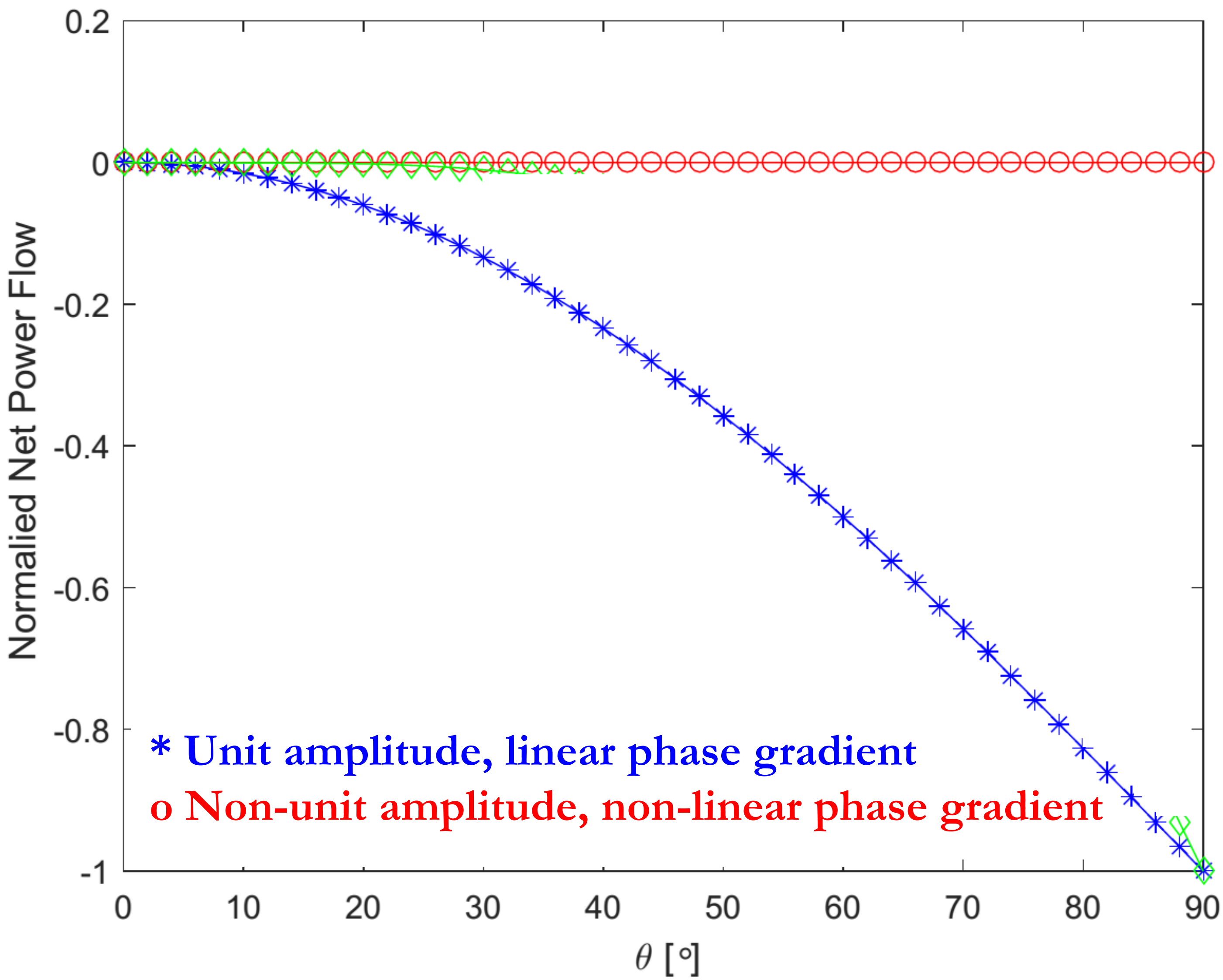
$$\sin(\theta_r) - \sin(\theta_i) = \frac{1}{n_i k_0}\frac{d\Phi}{dx}$$

Proof:
Fermat's Principle (1657)

"Light propagates from one point to another on trajectories such that the travel time is minimized"

Generalized Snell's Law:
How About the Reflected Power?

Amount of Reflected Power

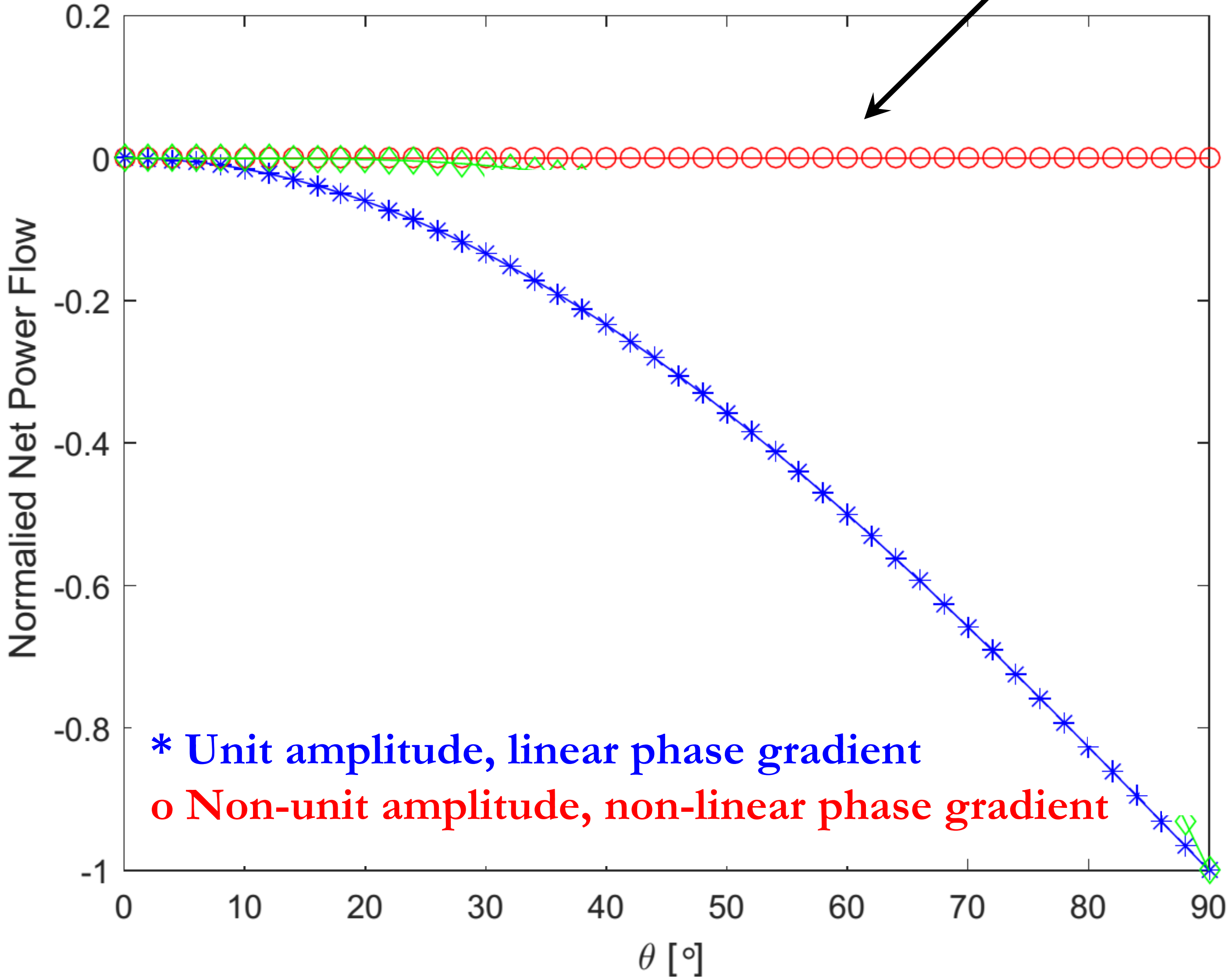


$$A\left(\theta^{(\text{incident})}, \theta^{(\text{reflected})}\right) = A_r = 1$$

$$A\left(\theta^{(\text{incident})}, \theta^{(\text{reflected})}\right) = A_{\text{opt}} = \sqrt{\frac{\cos\left(\theta^{(\text{incident})}\right)}{\cos\left(\theta^{(\text{reflected})}\right)}}$$

Amount of Reflected Power

Non-local sub-wavelength designs are needed



*** Unit amplitude, linear phase gradient**

o Non-unit amplitude, non-linear phase gradient

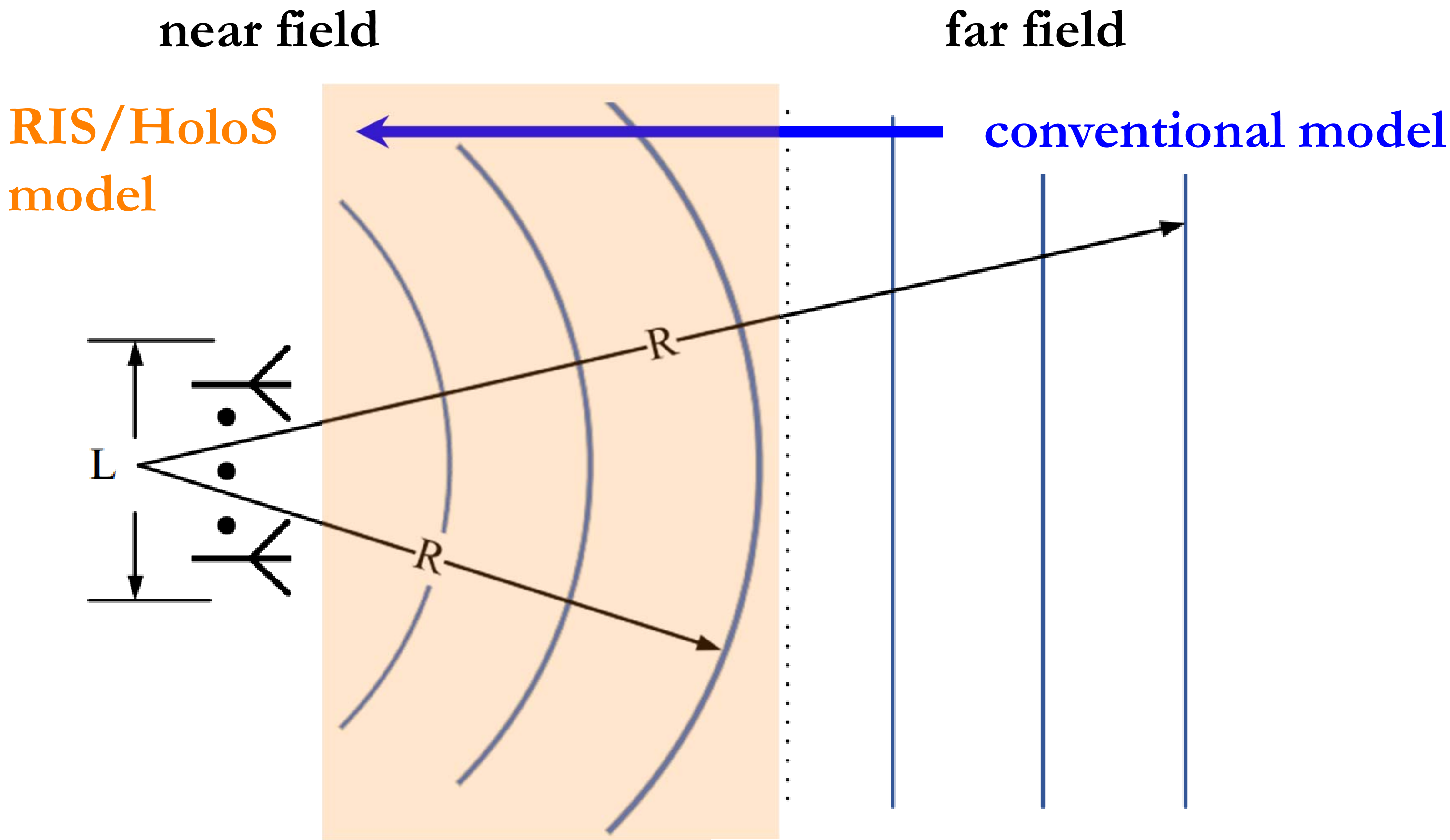
$$A\left(\theta^{(\text{incident})}, \theta^{(\text{reflected})}\right) = A_r = 1$$

$$A\left(\theta^{(\text{incident})}, \theta^{(\text{reflected})}\right) = A_{\text{opt}} = \sqrt{\frac{\cos\left(\theta^{(\text{incident})}\right)}{\cos\left(\theta^{(\text{reflected})}\right)}}$$

RIS/HoloS:
Rethinking the Communication Model

From Plane Waves for Spherical Waves

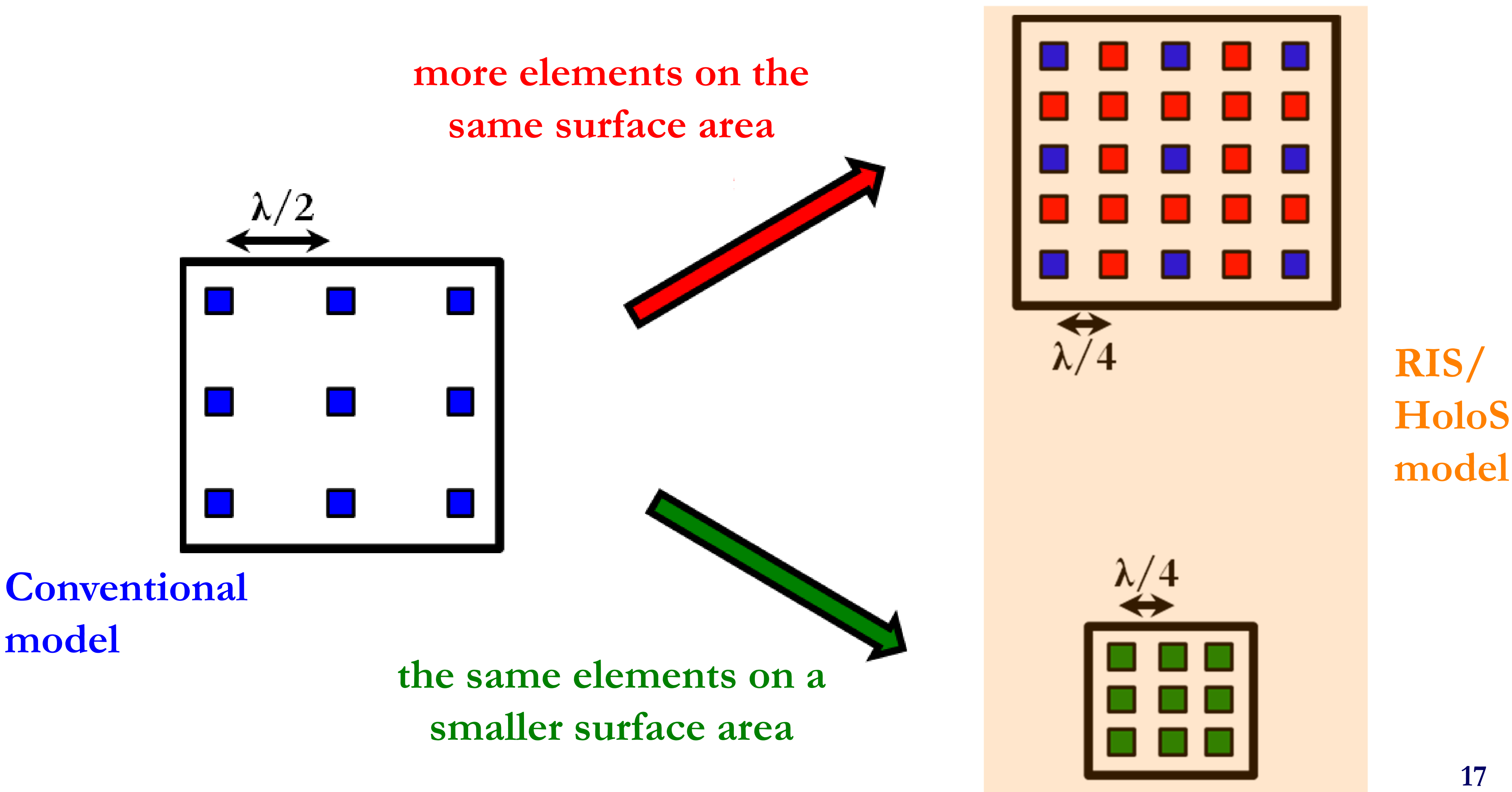
- **Paradigm #1: The wavefronts of the electromagnetic waves are (approximated as) locally planar on the antenna arrays**
  - **RISs/HoloS are electrically large and the transmission distances are shrinking**



$$R \geq R_f = \frac{2L^2}{\lambda}$$

Densification of Radiating Elements

- **Paradigm #2: The radiating elements of antenna-arrays are decoupled electromagnetically**
  - The inter-distances are smaller than the wavelength (< λ/2)



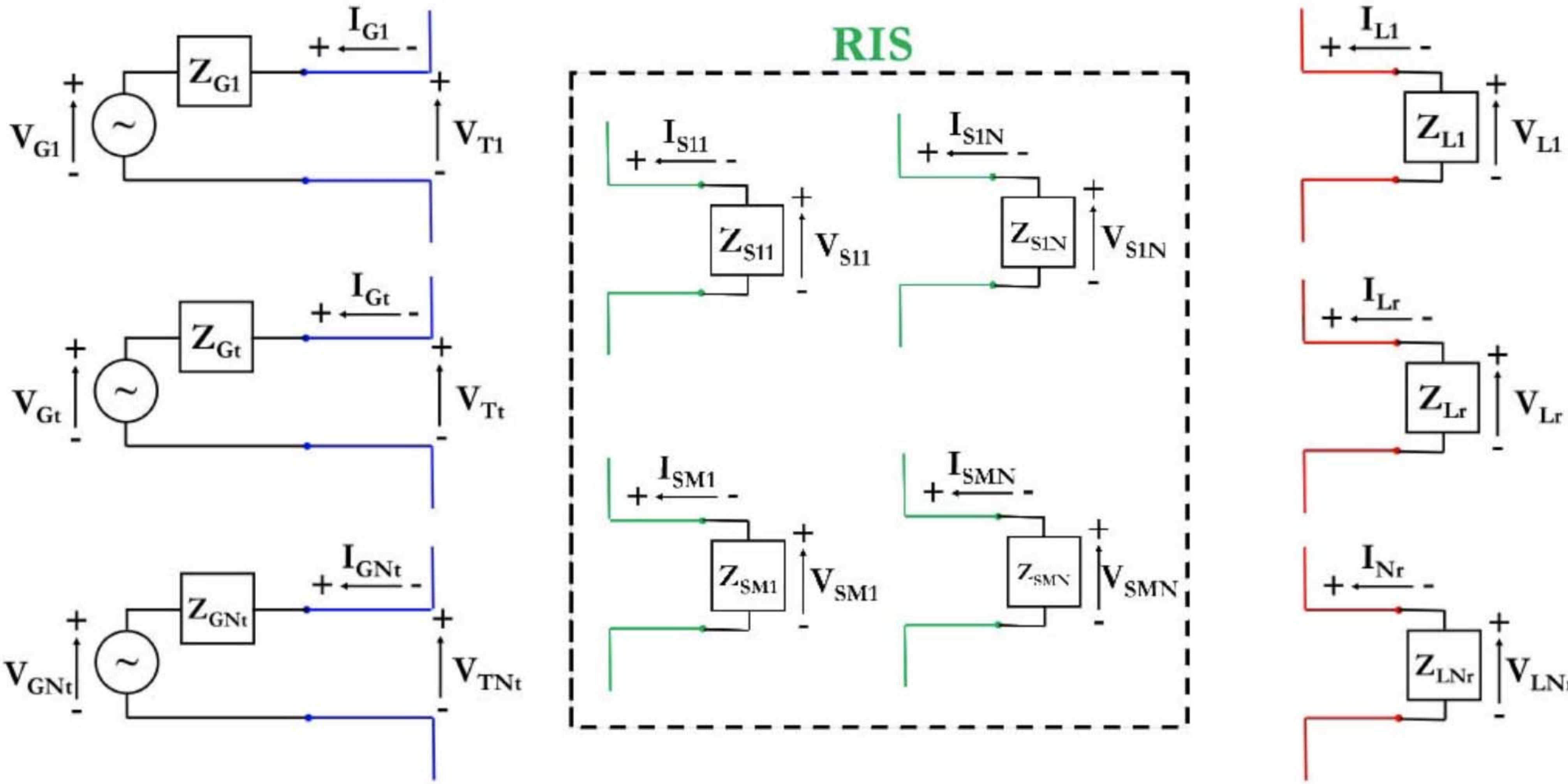
RIS:
A Loaded Thin Wire Model

RIS: A Loaded Thin Wire Model

- M. Di Renzo et al. "End-to-end mutual coupling aware communication model for reconfigurable intelligent surfaces: An electromagnetic-compliant approach based on mutual impedances", IEEE WCL 2021.
- M. Di Renzo et al. "Mutual coupling and unit cell aware optimization for reconfigurable intelligent surfaces", IEEE WCL 2021.
- M. Di Renzo et al. "MIMO interference channels assisted by reconfigurable intelligent surfaces: Mutual coupling aware sum-rate optimization based on a mutual impedance channel model", IEEE WCL 2021.
- M. Di Renzo at al. "Modeling the mutual coupling of reconfigurable metasurfaces", IEEE EuCAP 2023.
- M. Di Renzo et al. "Modeling and optimization of reconfigurable intelligent surfaces in propagation environments with scattering objects", IEEE WCL 2023 (submitted, under review).
- M. Di Renzo et al. "Optimization of RIS-aided SISO systems based on a mutually coupled loaded wire dipole model", IEEE ASILOMAR 2023 (submitted - invited, under review).
- M. Di Renzo et al. "Optimization of RIS-aided MIMO – A mutually coupled loaded wire dipole model", IEEE WCL 2023 (submitted, under review).
- …

Modeling in Free Space

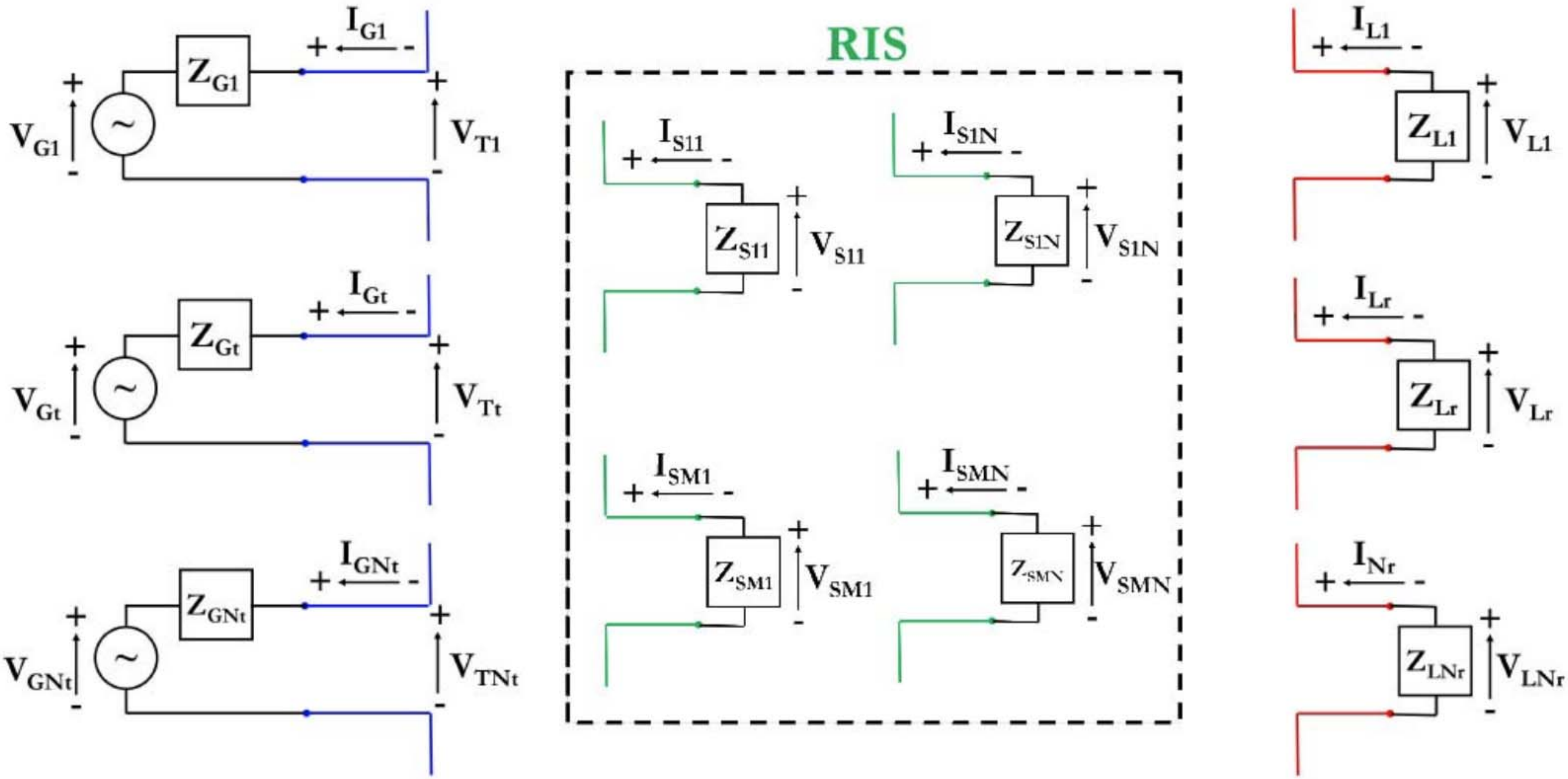
E2E Communication Model in Free Space



Discrete Dipole Approximation

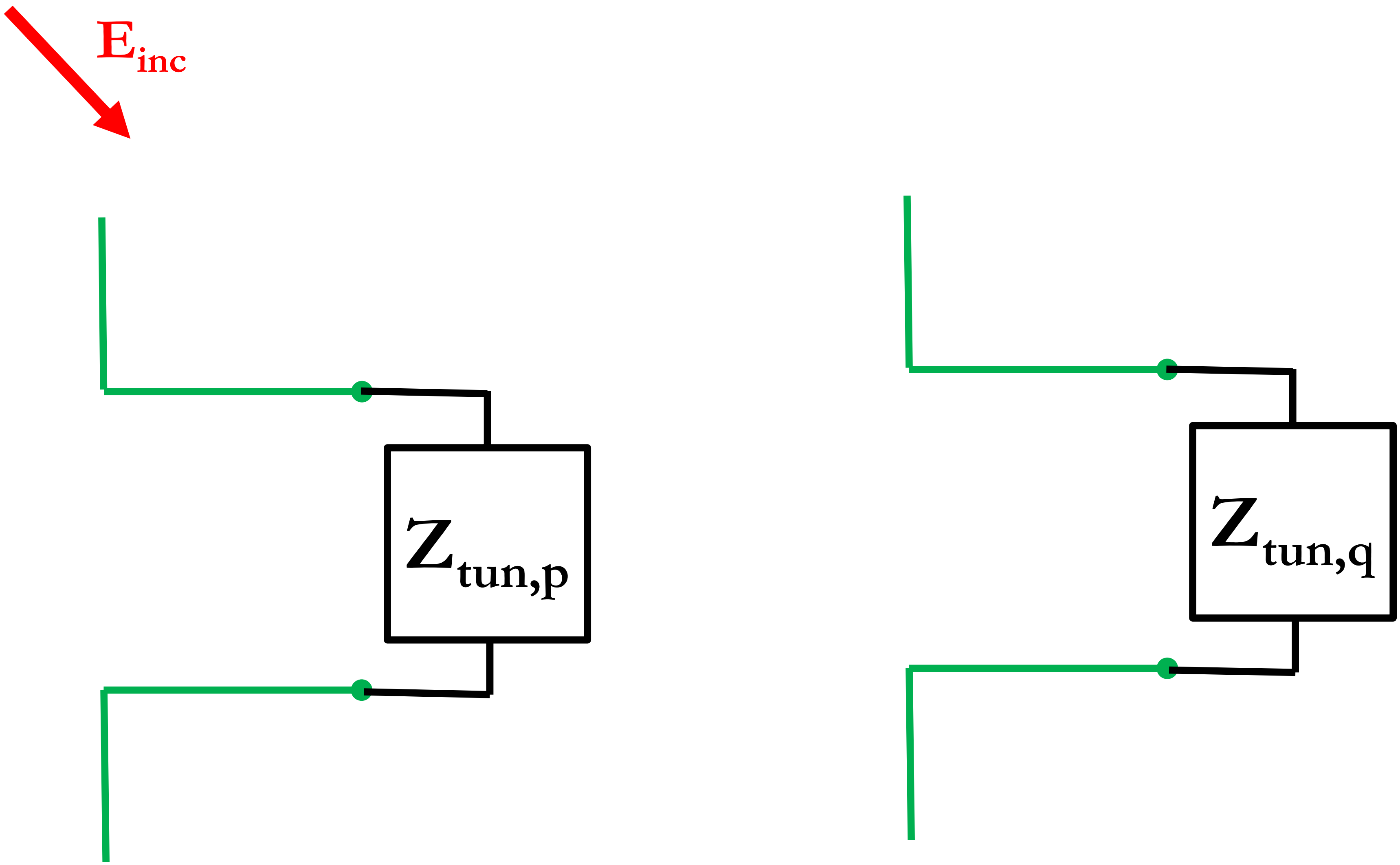
M. Johnson, P. Bowen, N. Kundtz, and A. Bily, "Discrete-dipole approximation model for control and optimization of a holographic metamaterial antenna", Appl. Opt., vol. 53, pp. 5791-5799, 2014.

E2E Communication Model in Free Space

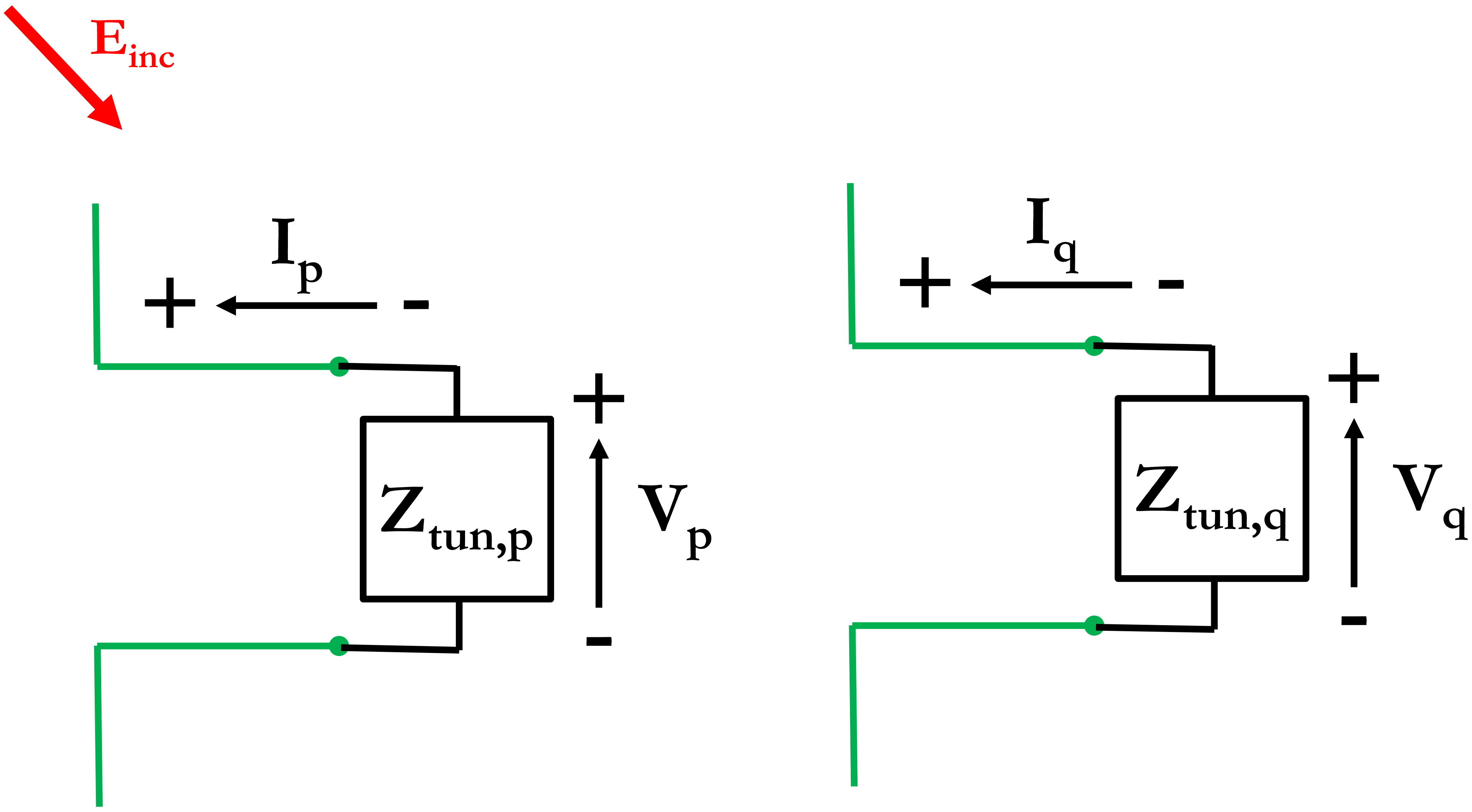


$$\mathbf{V}_{\mathrm{L}} = \mathcal{H}_{\mathrm{E2E}}\mathbf{V}_{\mathrm{G}}$$

Modeling the Mutual Coupling



Modeling the Mutual Coupling



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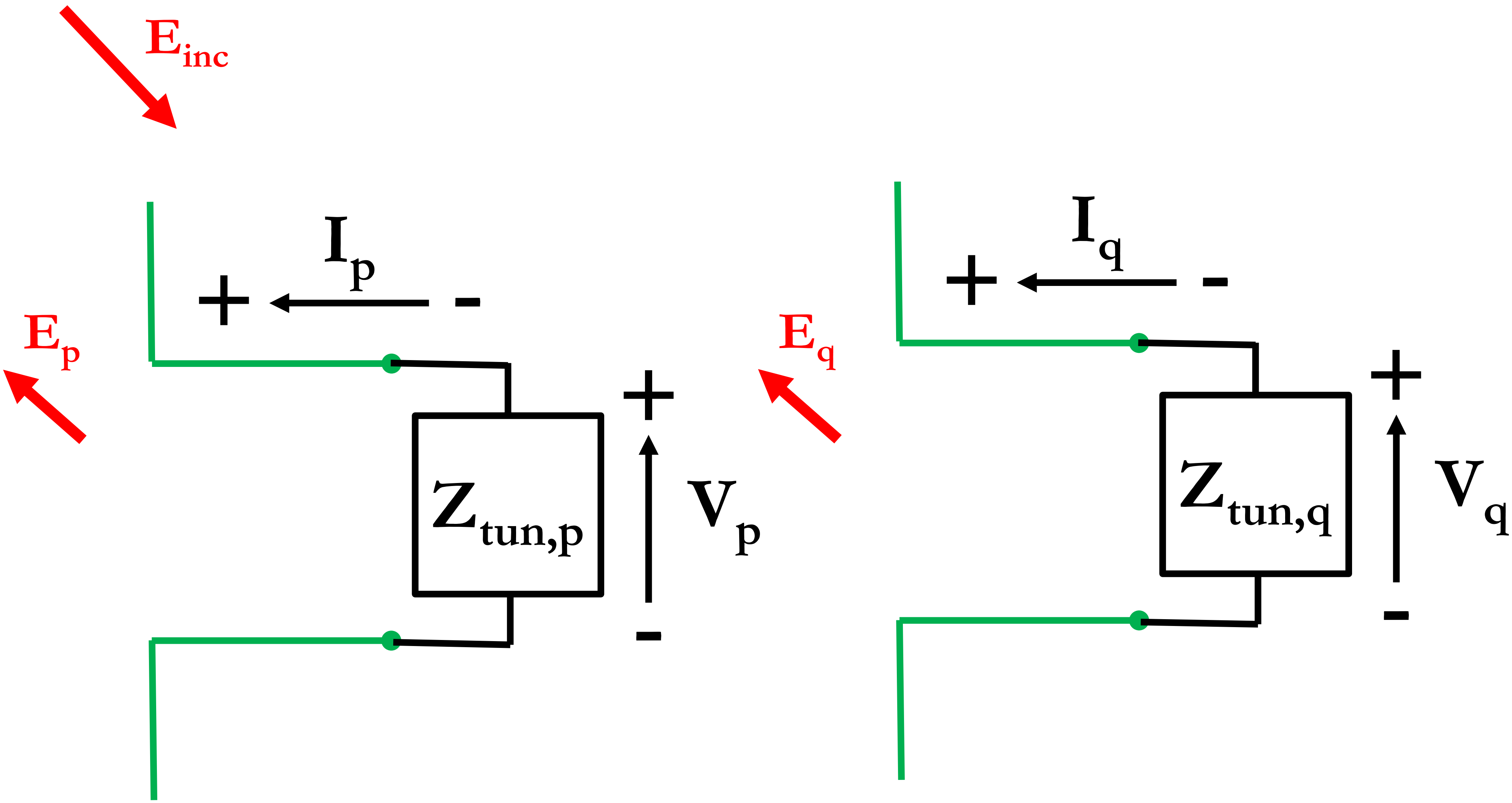

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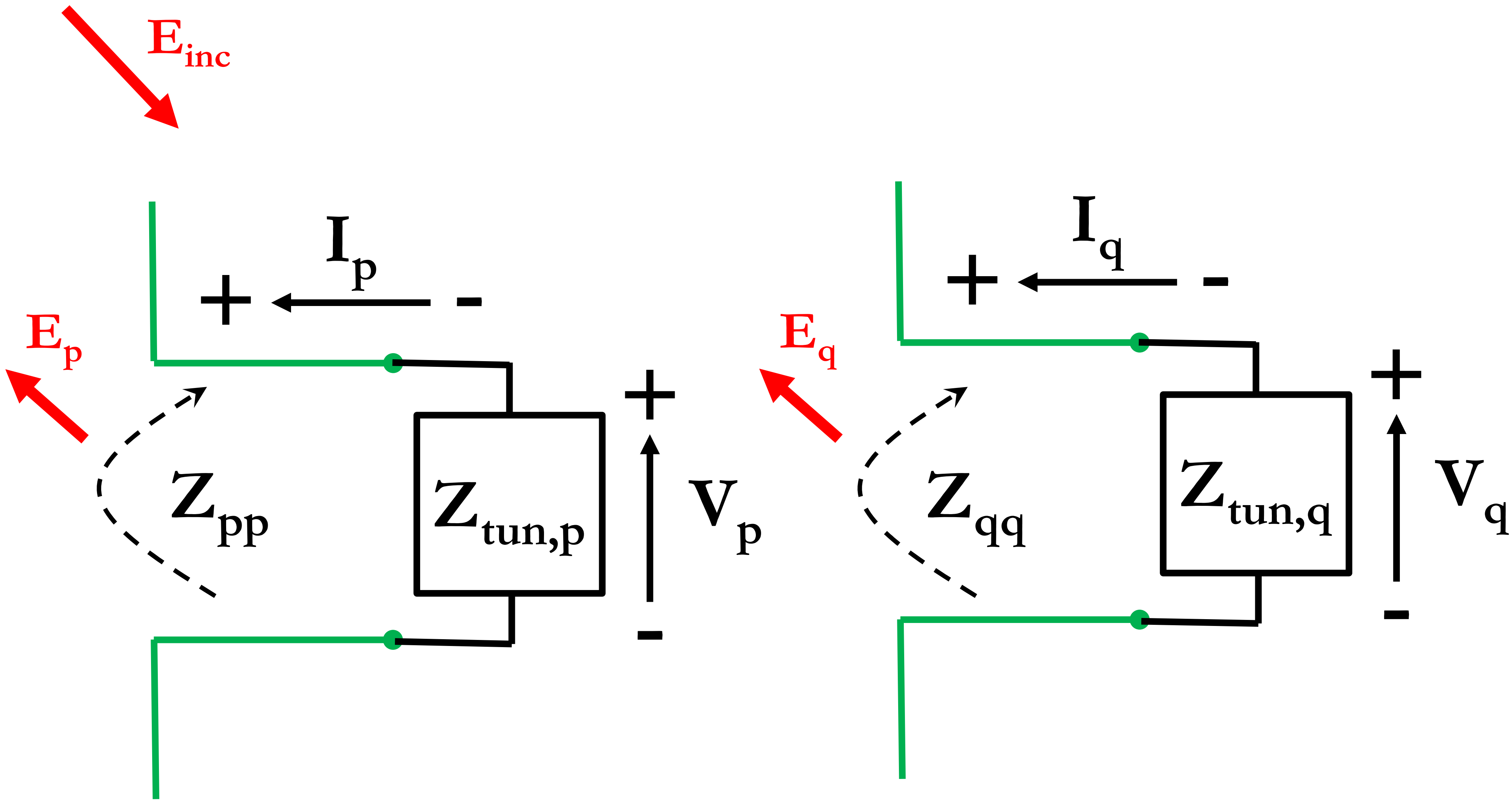

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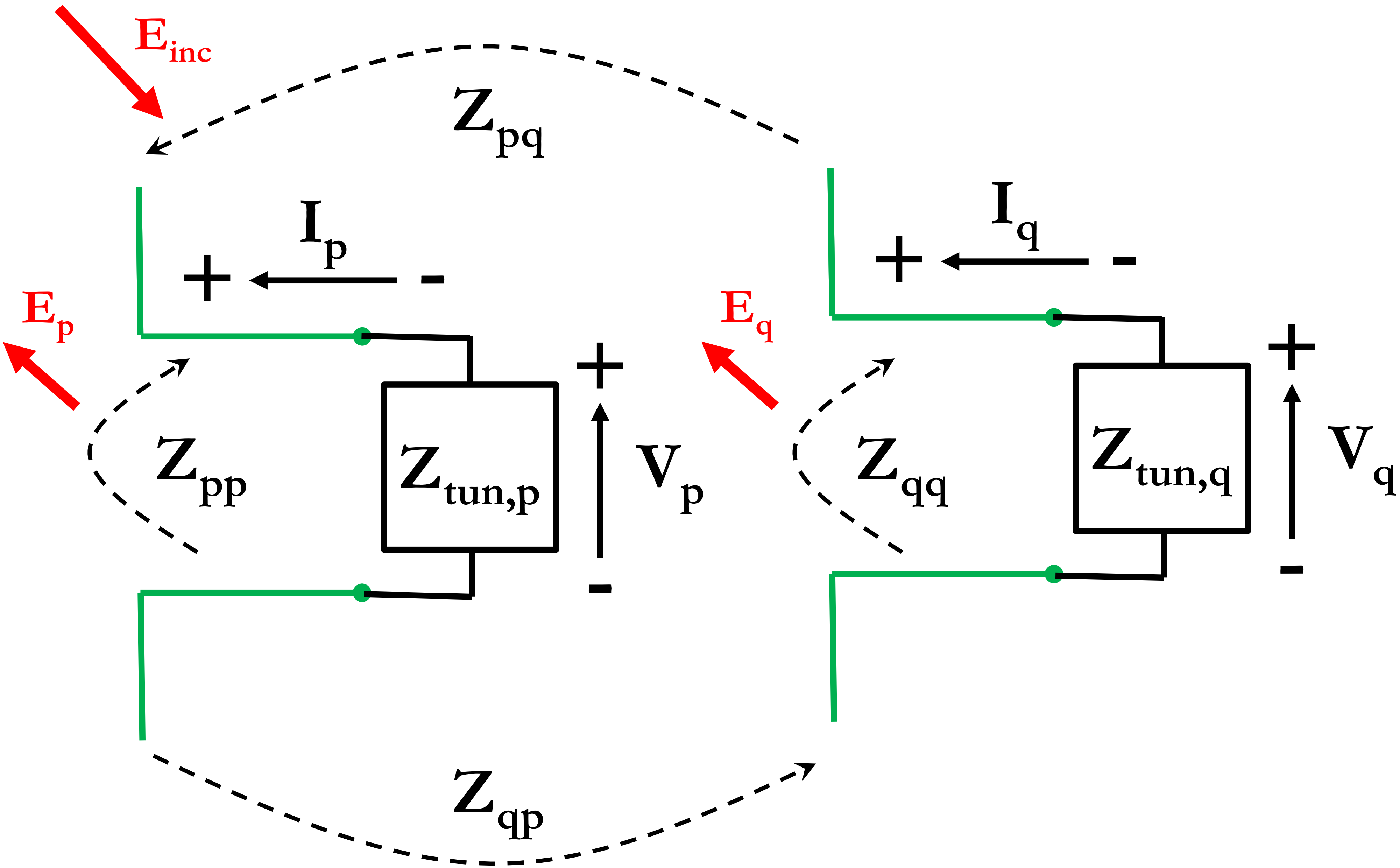
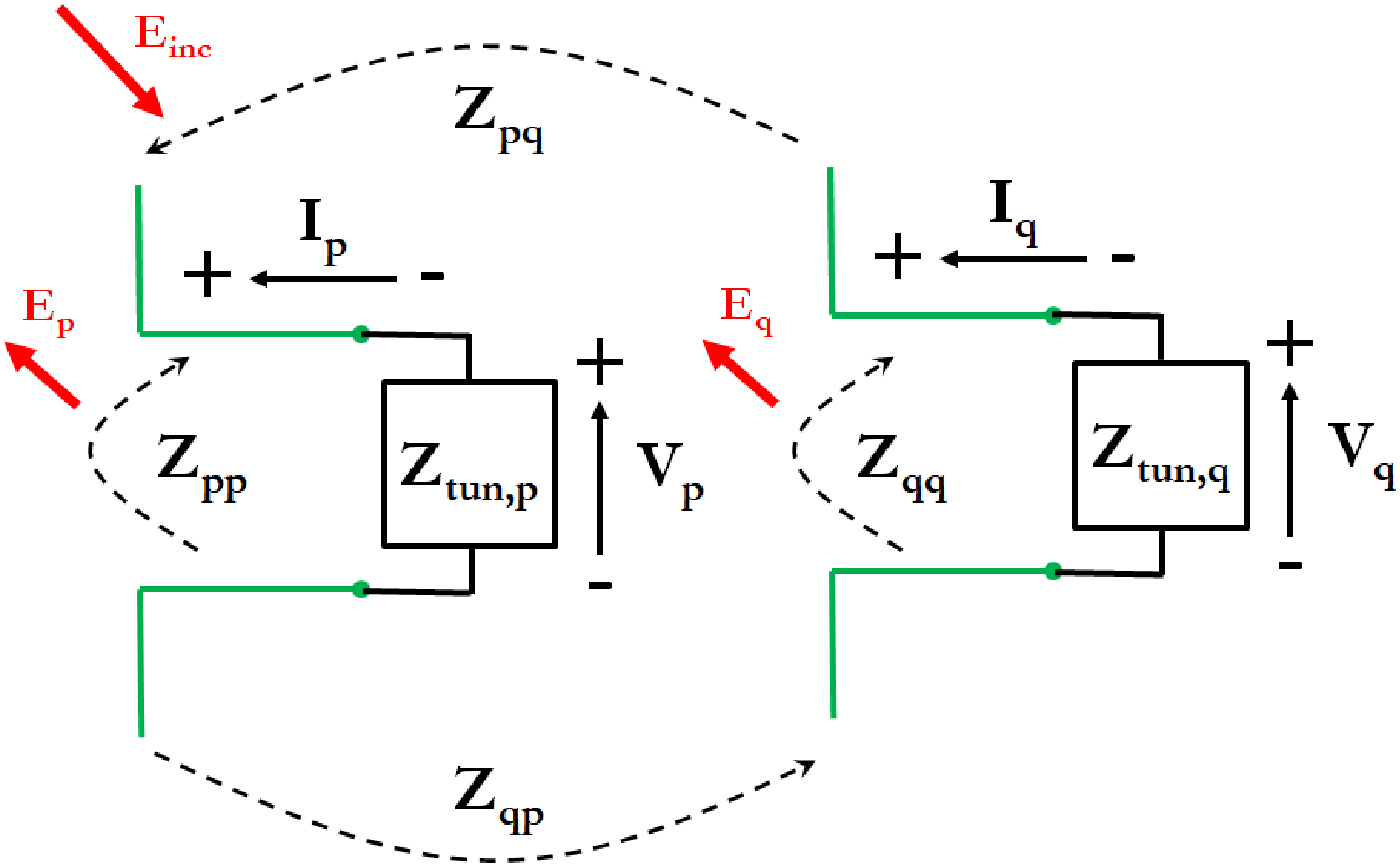
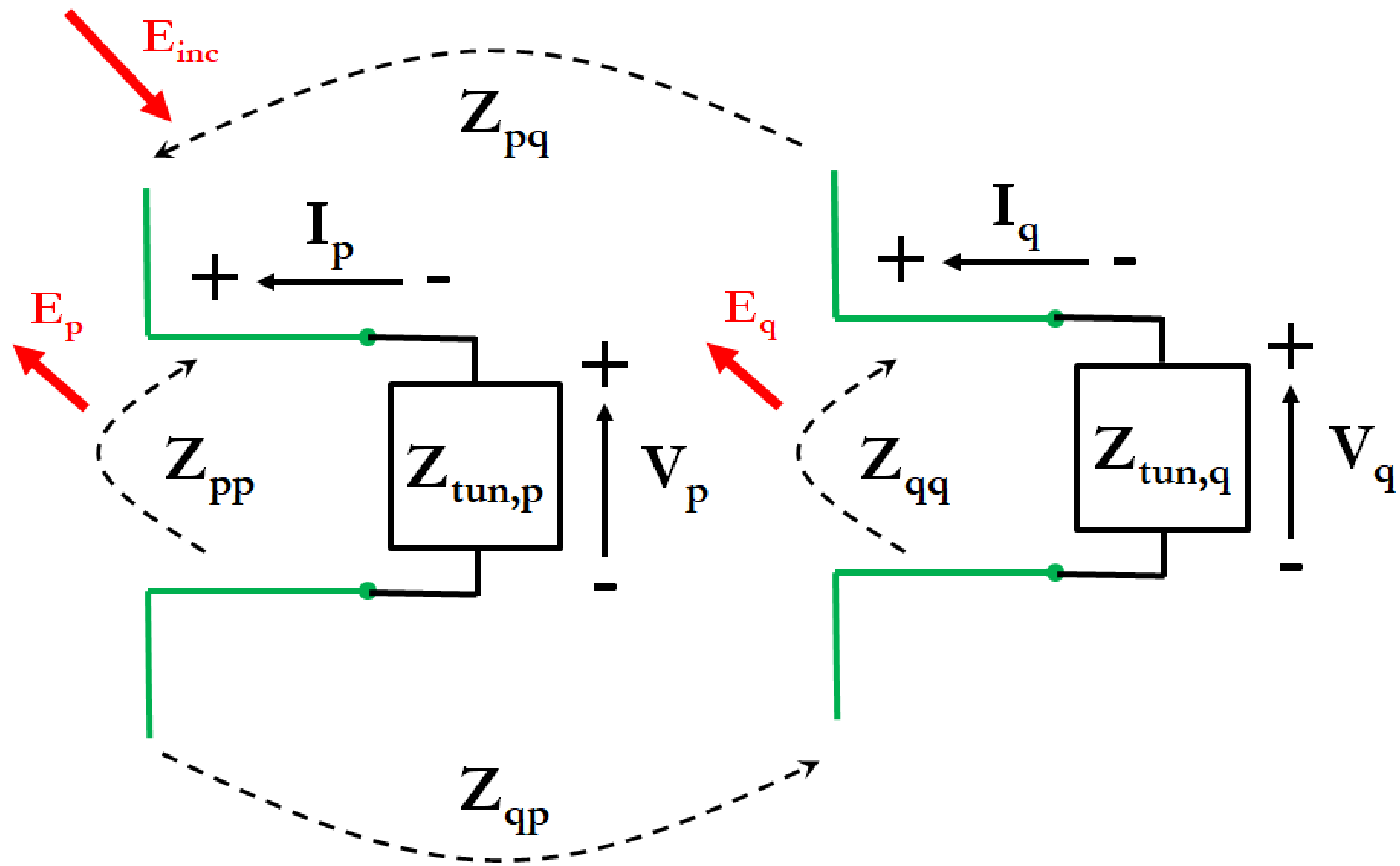


Modeling the Mutual Coupling



$$Z_{qp} = -\frac{1}{\mathcal{I}(z_q)\mathcal{I}(z_p)} \int_{z_q - l_q/2}^{z_q + l_q/2} E_{qp}^{(\mathrm{rad})}(z'') I_{z,q}(z'')\, dz''$$

Modeling the Mutual Coupling



Assumption:

Minimum Scattering Antenna

$$Z_{tun} \rightarrow \infty \quad \Rightarrow \quad I \rightarrow 0$$

$$\Downarrow$$

the antenna is "invisible"

Mutual Coupling: E2E Modeling

Mutual Coupling: E2E Modeling

- **Boundary conditions (SISO case)**

$$E_{qT}\left(z+z_q\right)+E_{qR}\left(z+z_q\right)+\sum_{n=1}^{N}E_{qSn}\left(z+z_q\right)=-V_q\left(z_q\right)\delta\left(z\right)$$

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- **Voltages at the ports of the thin wire dipoles**

$$\text{Transmiter: } V_G\left(z_T\right)-Z_G I_T\left(z_T\right)=V_T\left(z_T\right)$$

$$\text{Receiver: } -Z_R I_R\left(z_R\right)=V_R\left(z_R\right)$$

$$\text{RIS: } -Z_S I_S\left(z_S\right)=V_S\left(z_S\right), \quad S=\left\{S1,S2,\ldots,SN\right\}$$

Mutual Coupling: E2E Modeling

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- **Projection of the field on the currents and integration**

Mutual Coupling: E2E Modeling

- **A system of linear equations (2 RIS elements for simplicity)**

Mutual Coupling: E2E Modeling

- A system of linear equations (2 RIS elements for simplicity)

$$Z_{TT}I_T\left(z_T\right)+Z_{TR}I_R\left(z_R\right)+Z_{TS1}I_{S1}\left(z_{S1}\right)+Z_{TS2}I_{S2}\left(z_{S2}\right)=V_T\left(z_T\right)=V_G\left(z_T\right)-Z_GI_T\left(z_T\right)$$

$$Z_{RT}I_T\left(z_T\right)+Z_{RR}I_R\left(z_R\right)+Z_{RS1}I_{S1}\left(z_{S1}\right)+Z_{RS2}I_{S2}\left(z_{S2}\right)=V_R\left(z_R\right)=-Z_RI_R\left(z_R\right)$$

$$Z_{S1T}I_T\left(z_T\right)+Z_{S1R}I_R\left(z_R\right)+Z_{S1S1}I_{S1}\left(z_{S1}\right)+Z_{S1S2}I_{S2}\left(z_{S2}\right)=V_{S1}\left(z_{S1}\right)=-Z_{S1}I_{S1}\left(z_{S1}\right)$$

$$Z_{S2T}I_T\left(z_T\right)+Z_{S2R}I_R\left(z_R\right)+Z_{S2S1}I_{S1}\left(z_{S1}\right)+Z_{S2S2}I_{S2}\left(z_{S2}\right)=V_{S1}\left(z_{S1}\right)=-Z_{S2}I_{S2}\left(z_{S2}\right)$$

Mutual Coupling: E2E Modeling

- A system of linear equations (2 RIS elements for simplicity)

$$Z_{TT} I_T(z_T) + Z_{TR} I_R(z_R) + Z_{TS1} I_{S1}(z_{S1}) + Z_{TS2} I_{S2}(z_{S2}) = V_T(z_T) = V_G(z_T) - Z_G I_T(z_T)$$

$$Z_{RT} I_T(z_T) + Z_{RR} I_R(z_R) + Z_{RS1} I_{S1}(z_{S1}) + Z_{RS2} I_{S2}(z_{S2}) = V_R(z_R) = -Z_R I_R(z_R)$$

$$Z_{S1T} I_T(z_T) + Z_{S1R} I_R(z_R) + Z_{S1S1} I_{S1}(z_{S1}) + Z_{S1S2} I_{S2}(z_{S2}) = V_{S1}(z_{S1}) = -Z_{S1} I_{S1}(z_{S1})$$

$$Z_{S2T} I_T(z_T) + Z_{S2R} I_R(z_R) + Z_{S2S1} I_{S1}(z_{S1}) + Z_{S2S2} I_{S2}(z_{S2}) = V_{S1}(z_{S1}) = -Z_{S2} I_{S2}(z_{S2})$$

Mutual Coupling: E2E Modeling

- **A system of linear equations (2 RIS elements for simplicity)**

$$Z_{TT}I_T(z_T)+Z_{TR}I_R(z_R)+Z_{TS1}I_{S1}(z_{S1})+Z_{TS2}I_{S2}(z_{S2})=V_T(z_T)=V_G(z_T)-Z_GI_T(z_T)$$

$$Z_{RT}I_T(z_T)+Z_{RR}I_R(z_R)+Z_{RS1}I_{S1}(z_{S1})+Z_{RS2}I_{S2}(z_{S2})=V_R(z_R)=-Z_RI_R(z_R)$$

$$Z_{S1T}I_T(z_T)+Z_{S1R}I_R(z_R)+Z_{S1S1}I_{S1}(z_{S1})+Z_{S1S2}I_{S2}(z_{S2})=V_{S1}(z_{S1})=-Z_{S1}I_{S1}(z_{S1})$$

$$Z_{S2T}I_T(z_T)+Z_{S2R}I_R(z_R)+Z_{S2S1}I_{S1}(z_{S1})+Z_{S2S2}I_{S2}(z_{S2})=V_{S1}(z_{S1})=-Z_{S2}I_{S2}(z_{S2})$$

- **E2E channel**

$$H_{e2e}(Z_{S1},Z_{S2})=\frac{V_L}{V_G}$$

$$=\frac{1}{\left(Z_G+Z_{TT}+\Psi_{TT}(Z_{S1},Z_{S2})\right)\left(Z_{RT}+\Psi_{RT}(Z_{S1},Z_{S2})\right)^{-1}\left(Z_L^{-1}\left(Z_{RR}+\Psi_{RR}(Z_{S1},Z_{S2})\right)+1\right)-Z_L^{-1}\left(Z_{TR}+\Psi_{TR}(Z_{S1},Z_{S2})\right)}$$

Mutual Coupling: E2E Modeling

Theorem 1: Let $\mathcal{Z}_{\mathrm{XY}}$ be the $\mathrm{N_x} \times \mathrm{N_y}$ matrix whose (x, y)th entry is the mutual impedance between the xth and yth radiating elements of X and Y, where $\mathrm{X}, \mathrm{Y} = \{\mathrm{T}, \mathrm{S}, \mathrm{R}\}$ and $\mathrm{N_x}, \mathrm{N_y} = \{\mathrm{N_t}, \mathrm{N_{ris}}, \mathrm{N_r}\}$, and T, S, R identify the transmitter, the RIS, and the receiver, respectively. $\mathcal{H}_{\mathrm{E2E}}$ is as follows:

$$\mathcal{H}_{\mathrm{E2E}} = \left(\mathbf{I}_{N_{\mathrm{r}} \times N_{\mathrm{r}}} + \mathcal{P}_{\mathrm{RSR}} \mathbf{Z}_{\mathrm{L}}^{-1} - \mathcal{P}_{\mathrm{RST}} \mathcal{P}_{\mathrm{GTST}}^{-1} \mathcal{P}_{\mathrm{TSR}} \mathbf{Z}_{\mathrm{L}}^{-1}\right)^{-1} \times \mathcal{P}_{\mathrm{RST}} \mathcal{P}_{\mathrm{GTST}}^{-1}$$

where $\mathbf{I}_{N_{\mathrm{r}} \times N_{\mathrm{r}}}$ denotes an $N_{\mathrm{r}} \times N_{\mathrm{r}}$ identity matrix, $\mathbf{Z}_{\mathrm{G}}$ is the $N_{\mathrm{t}} \times N_{\mathrm{t}}$ diagonal matrix whose (t, t)th entry is Z_{Gt}, $\mathbf{Z}_{\mathrm{RIS}}$ is the $N_{\mathrm{ris}} \times N_{\mathrm{ris}}$ diagonal matrix whose (mn, mn)th entry is Z_{Smn}, $\mathbf{Z}_{\mathrm{L}}$ is the $N_{\mathrm{r}} \times N_{\mathrm{r}}$ diagonal matrix whose (r, r)th entry is Z_{Lr}, $\mathcal{P}_{\mathrm{GTST}} = \mathbf{Z}_{\mathrm{G}} + \mathcal{P}_{\mathrm{TST}}$, and:

$$\mathcal{P}_{\mathrm{XSY}} = \mathcal{Z}_{\mathrm{XY}} - \mathcal{Z}_{\mathrm{XS}} \left(\mathbf{Z}_{\mathrm{RIS}} + \mathcal{Z}_{\mathrm{SS}}\right)^{-1} \mathcal{Z}_{\mathrm{SY}}$$

Mutual Coupling: E2E Modeling

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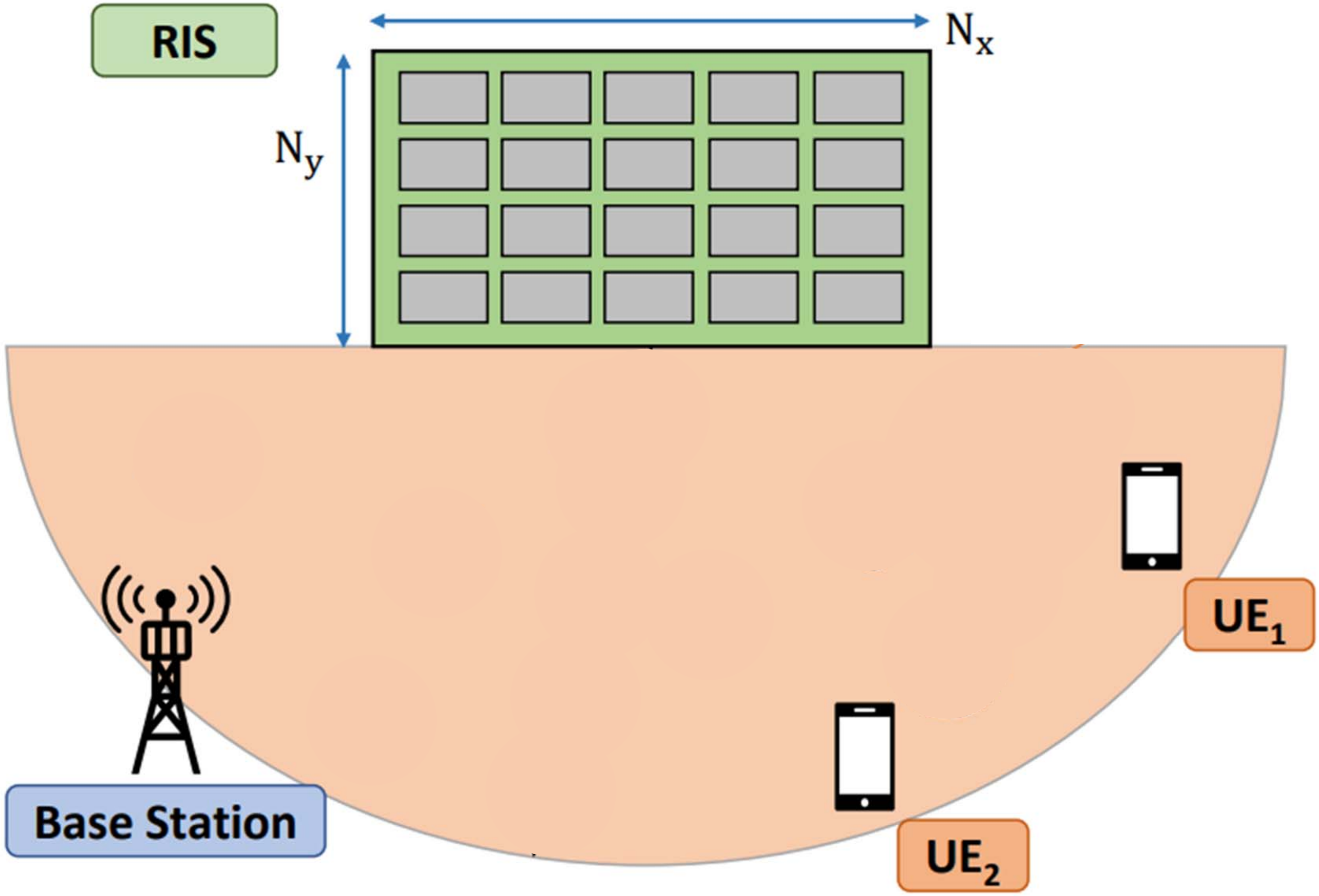
Tunable loads / Circuit model of the RIS

Mutual coupling of the RIS

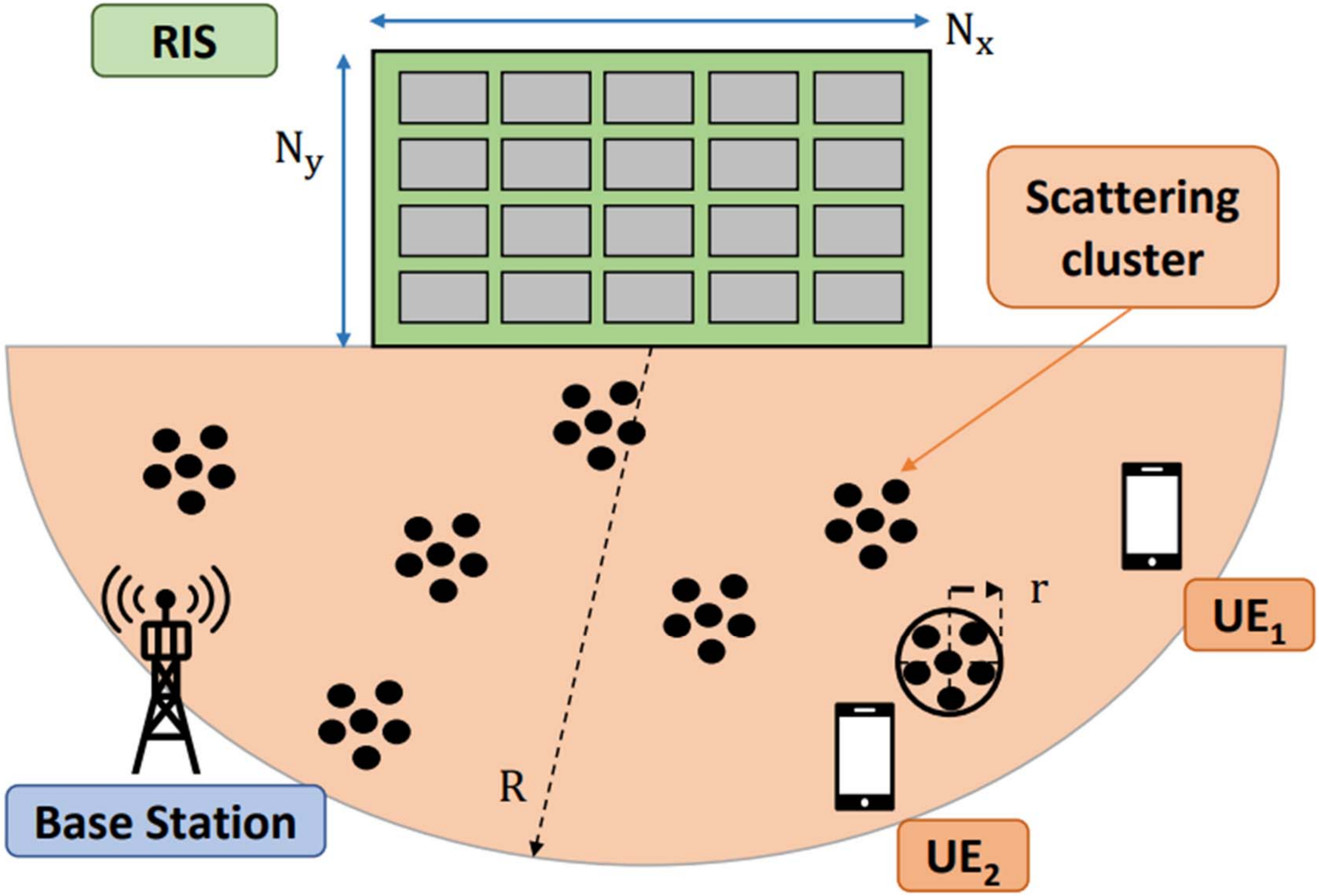
$$\mathcal{P}_{\mathrm{XSY}} = \mathcal{Z}_{\mathrm{XY}} - \mathcal{Z}_{\mathrm{XS}} \left(\boxed{\mathbf{Z}_{\mathrm{RIS}}} + \boxed{\mathcal{Z}_{\mathrm{SS}}}\right)^{-1} \mathcal{Z}_{\mathrm{SY}}$$

Modeling in the Presence of Scattering Objects (Multipath)

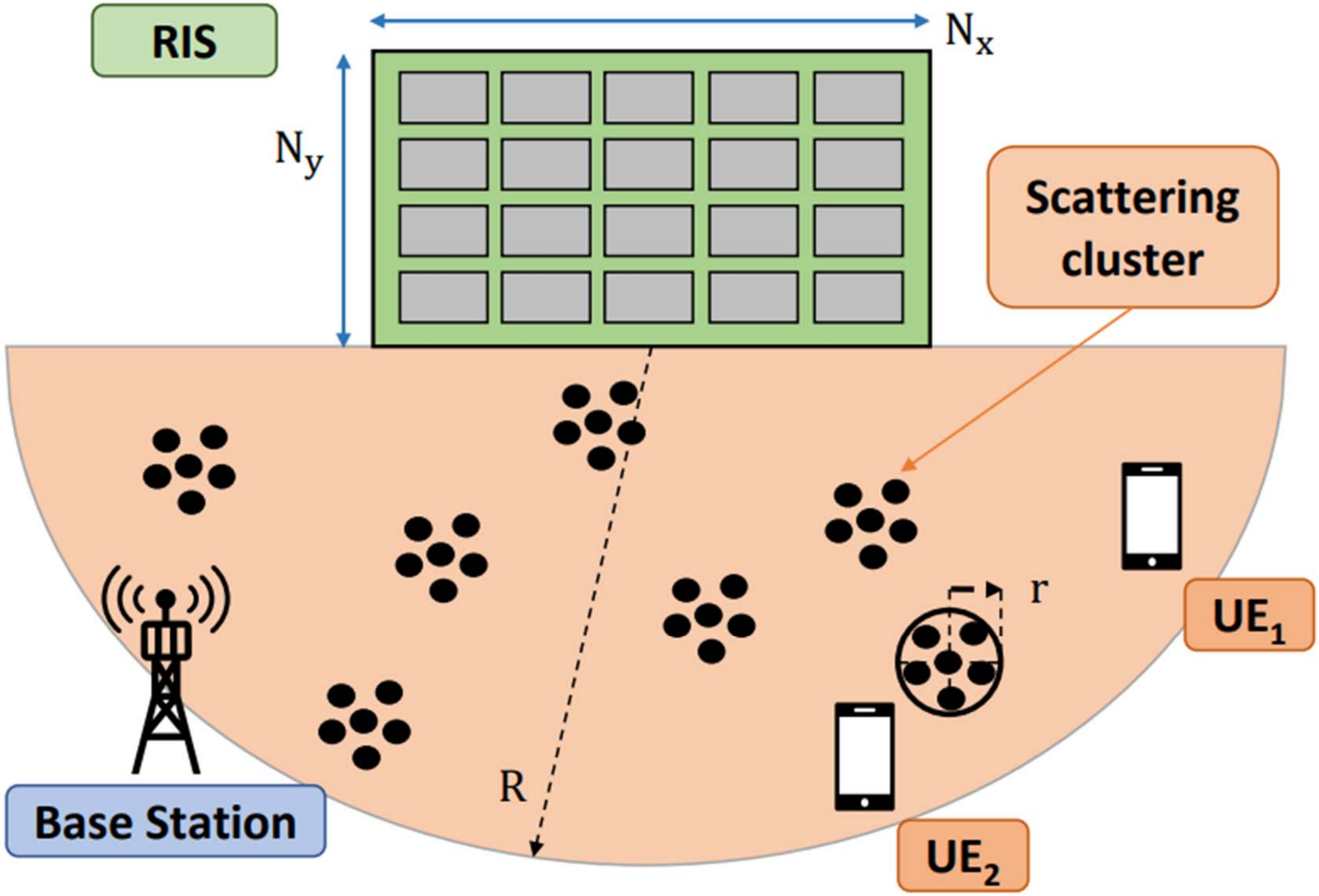
Modeling in the Presence of Scattering Objects



Modeling in the Presence of Scattering Objects



Modeling in the Presence of Scattering Objects



$$\mathbf{V}_{\mathrm{L}} = \mathcal{H}_{\mathrm{E2E}} \mathbf{V}_{\mathrm{G}}$$

Modeling in the Presence of Scattering Objects

mathematics

MDPI

Review

The Discrete Dipole Approximation: A Review

Patrick Christian Chaumet

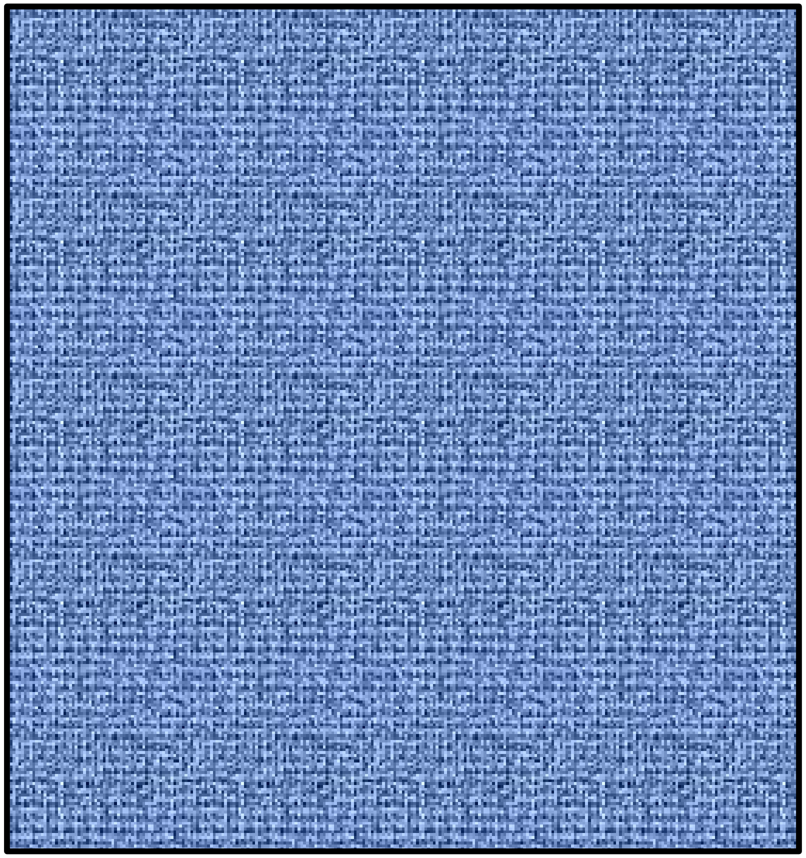
Institut Fresnel, Aix Marseille Univ, CNRS, Centrale Marseille, CEDEX 20, 13397 Marseille, France; patrick.chaumet@fresnel.fr

Abstract: There are many methods for rigorously calculating electromagnetic diffraction by objects of arbitrary shape and permittivity. In this article, we will detail the discrete dipole approximation (DDA) which belongs to the class of volume integral methods. Starting from Maxwell's equations, we will first present the principle of DDA as well as its theoretical and numerical aspects. Then, we will discuss the many developments that this method has undergone over time and the numerous applications that have been developed to transform DDA in a very versatile method. We conclude with a discussion of the strengths and weaknesses of the DDA and a description of the freely available DDA-based electromagnetic diffraction codes.

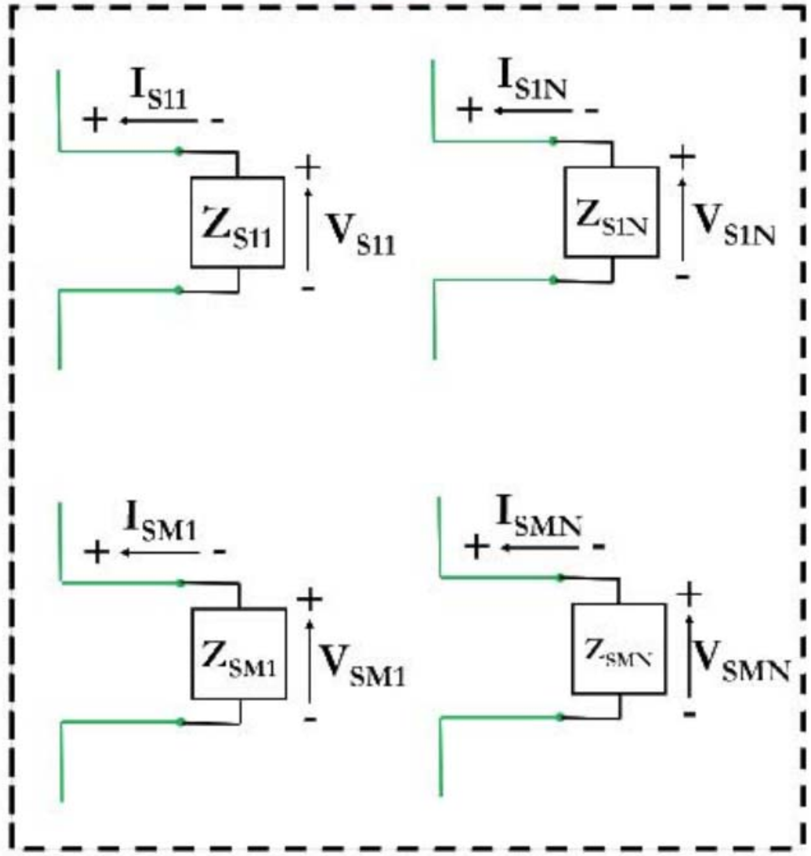
Keywords: electromagnetic simulation; DDA; numerical method; electromagnetic scattering

Modeling in the Presence of Scattering Objects

material object



discrete dipole approx.



the properties of the dipoles (length, radius, etc.) and the load impedances depend on the material object being considered

Modeling in the Presence of Scattering Objects

$$\mathbf{H}_{\mathrm{E2E}} = \left(\mathbf{I}_L + \mathbf{Z}_{\mathrm{RR}}\mathbf{Z}_{\mathrm{L}}^{-1}\right)^{-1}\left[\mathbf{Z}_{\mathrm{RT}} - \mathbf{Z}_{\mathrm{RE}}\left(\mathbf{Z}_{\mathrm{EE}} + \mathbf{Z}_{\mathrm{SC}}\right)^{-1}\mathbf{Z}_{\mathrm{ET}}\right] \times \left(\mathbf{Z}_{\mathrm{TT}} + \mathbf{Z}_{\mathrm{G}}\right)^{-1} \in \mathbb{C}^{L \times M}$$

$\mathbf{Z}_{\mathrm{ET}}$: Impedances between Tx and scatterers (RIS & objects)

$\mathbf{Z}_{\mathrm{RE}}$: Impedances between scatterers (RIS & objects) and Rx

$\mathbf{Z}_{\mathrm{EE}}$: Impedances between scatterers (RIS & objects)

$\mathbf{Z}_{\mathrm{SC}}$: Tunable loads of RIS and material impedances of scatterers

$$\mathbf{Z}_{\mathrm{EE}} = \begin{bmatrix} \mathbf{Z}_{\mathrm{OO}} & \mathbf{Z}_{\mathrm{OS}} \\ \mathbf{Z}_{\mathrm{SO}} & \mathbf{Z}_{\mathrm{SS}} \end{bmatrix} \qquad \mathbf{Z}_{\mathrm{SC}} = \begin{bmatrix} \mathbf{Z}_{\mathrm{US}} & \mathbf{0}_{N_s \times N} \\ \mathbf{0}_{N \times N_s} & \mathbf{Z}_{\mathrm{RIS}} \end{bmatrix}$$

Modeling in the Presence of Scattering Objects

$$\mathbf{H}_{\text{E2E}} = \left(\mathbf{I}_L + \mathbf{Z}_{\text{RR}}\mathbf{Z}_{\text{L}}^{-1}\right)^{-1} \left[\mathbf{Z}_{\text{RT}} - \mathbf{Z}_{\text{RE}}\left(\mathbf{Z}_{\text{EE}} + \mathbf{Z}_{\text{SC}}\right)^{-1}\mathbf{Z}_{\text{ET}}\right] \times \left(\mathbf{Z}_{\text{TT}} + \mathbf{Z}_{\text{G}}\right)^{-1} \in \mathbb{C}^{L \times M}$$

Schur complement to block matrices

$$\mathbf{H}_{\text{E2E}} = \mathbf{Z}_{\text{RL}}\left[\mathbf{Z}_{\text{ROT}} - \mathbf{Z}_{\text{ROS}}\left(\mathbf{Z}_{\text{SS}} + \mathbf{Z}_{\text{SOS}} + \mathbf{Z}_{\text{RIS}}\right)^{-1}\mathbf{Z}_{\text{SOT}}\right]\mathbf{Z}_{\text{TG}}$$

The obtained model is formally equivalent to free space: $\mathbf{Z}_{\text{RIS}}$ is decoupled from the matrices of mutual coupling

Modeling in the Presence of Scattering Objects

$$\mathbf{H}_{\mathrm{E2E}} = \left(\mathbf{I}_L + \mathbf{Z}_{\mathrm{RR}}\mathbf{Z}_{\mathrm{L}}^{-1}\right)^{-1}\left[\mathbf{Z}_{\mathrm{RT}} - \mathbf{Z}_{\mathrm{RE}}\left(\mathbf{Z}_{\mathrm{EE}} + \mathbf{Z}_{\mathrm{SC}}\right)^{-1}\mathbf{Z}_{\mathrm{ET}}\right] \times \left(\mathbf{Z}_{\mathrm{TT}} + \mathbf{Z}_{\mathrm{G}}\right)^{-1} \in \mathbb{C}^{L\times M}$$

Schur complement to block matrices

$$\mathbf{H}_{\mathrm{E2E}} = \mathbf{Z}_{\mathrm{RL}}\left[\mathbf{Z}_{\mathrm{ROT}} - \mathbf{Z}_{\mathrm{ROS}}\left(\mathbf{Z}_{\mathrm{SS}} + \mathbf{Z}_{\mathrm{SOS}} + \mathbf{Z}_{\mathrm{RIS}}\right)^{-1}\mathbf{Z}_{\mathrm{SOT}}\right]\mathbf{Z}_{\mathrm{TG}}$$

Insights: The scatterers do not contribute as an additive term:

$$\mathbf{H}_{\mathrm{E2E}} \neq \mathbf{H}_{\mathrm{E2E}}\left(\text{free space}\right) + \mathbf{H}_{\mathrm{Multipath}}$$

Modeling in the Presence of Scattering Objects

$$\mathbf{H}_{\mathrm{E2E}} = \left(\mathbf{I}_L + \mathbf{Z}_{\mathrm{RR}}\mathbf{Z}_{\mathrm{L}}^{-1}\right)^{-1}\left[\mathbf{Z}_{\mathrm{RT}} - \mathbf{Z}_{\mathrm{RE}}\left(\mathbf{Z}_{\mathrm{EE}} + \mathbf{Z}_{\mathrm{SC}}\right)^{-1}\mathbf{Z}_{\mathrm{ET}}\right] \times \left(\mathbf{Z}_{\mathrm{TT}} + \mathbf{Z}_{\mathrm{G}}\right)^{-1} \in \mathbb{C}^{L \times M}$$

Schur complement to block matrices

$$\mathbf{H}_{\mathrm{E2E}} = \mathbf{Z}_{\mathrm{RL}}\left[\mathbf{Z}_{\mathrm{ROT}} - \mathbf{Z}_{\mathrm{ROS}}\left(\mathbf{Z}_{\mathrm{SS}} + \mathbf{Z}_{\mathrm{SOS}} + \mathbf{Z}_{\mathrm{RIS}}\right)^{-1}\mathbf{Z}_{\mathrm{SOT}}\right]\mathbf{Z}_{\mathrm{TG}}$$

Insights: But, if $Z_{SO} = 0$ and $Z_{OS} = 0$, then

$$\mathbf{H}_{\mathrm{E2E}} = \mathbf{Z}_{\mathrm{RL}}\left[\mathbf{Z}_{\mathrm{RT}} - \mathbf{Z}_{\mathrm{RO}}\overline{\mathbf{Z}}_{\mathrm{OO}}^{-1}\mathbf{Z}_{\mathrm{OT}} - \mathbf{Z}_{\mathrm{RS}}\left(\mathbf{Z}_{\mathrm{SS}} + \mathbf{Z}_{\mathrm{RIS}}\right)^{-1}\mathbf{Z}_{\mathrm{ST}}\right]\mathbf{Z}_{\mathrm{TG}}$$

Modeling in the Presence of Scattering Objects

$$\mathbf{H}_{\mathrm{E2E}} = \left(\mathbf{I}_L + \mathbf{Z}_{\mathrm{RR}}\mathbf{Z}_{\mathrm{L}}^{-1}\right)^{-1}\left[\mathbf{Z}_{\mathrm{RT}} - \mathbf{Z}_{\mathrm{RE}}\left(\mathbf{Z}_{\mathrm{EE}} + \mathbf{Z}_{\mathrm{SC}}\right)^{-1}\mathbf{Z}_{\mathrm{ET}}\right] \times \left(\mathbf{Z}_{\mathrm{TT}} + \mathbf{Z}_{\mathrm{G}}\right)^{-1} \in \mathbb{C}^{L\times M}$$

Schur complement to block matrices

$$\mathbf{H}_{\mathrm{E2E}} = \mathbf{Z}_{\mathrm{RL}}\left[\mathbf{Z}_{\mathrm{ROT}} - \mathbf{Z}_{\mathrm{ROS}}\left(\mathbf{Z}_{\mathrm{SS}} + \mathbf{Z}_{\mathrm{SOS}} + \mathbf{Z}_{\mathrm{RIS}}\right)^{-1}\mathbf{Z}_{\mathrm{SOT}}\right]\mathbf{Z}_{\mathrm{TG}}$$

Insights: But, if $\mathbf{Z}_{\mathrm{SO}} = 0$ and $\mathbf{Z}_{\mathrm{OS}} = 0$, then

$$\mathbf{H}_{\mathrm{E2E}} = \mathbf{H}_{\mathrm{E2E}}\left(\text{free space}\right) + \mathbf{H}_{\mathrm{Multipath}}$$

Optimization

RIS: A Loaded Thin Wire Model

- M. Di Renzo et al. "End-to-end mutual coupling aware communication model for reconfigurable intelligent surfaces: An electromagnetic-compliant approach based on mutual impedances", IEEE WCL 2021.
- M. Di Renzo et al. "Mutual coupling and unit cell aware optimization for reconfigurable intelligent surfaces", IEEE WCL 2021.
- M. Di Renzo et al. "MIMO interference channels assisted by reconfigurable intelligent surfaces: Mutual coupling aware sum-rate optimization based on a mutual impedance channel model", IEEE WCL 2021.
- M. Di Renzo at al. "Modeling the mutual coupling of reconfigurable metasurfaces", IEEE EuCAP 2023.
- M. Di Renzo et al. "Modeling and optimization of reconfigurable intelligent surfaces in propagation environments with scattering objects", IEEE WCL 2023 (submitted, under review).
- M. Di Renzo et al. "Optimization of RIS-aided SISO systems based on a mutually coupled loaded wire dipole model", IEEE ASILOMAR 2023 (submitted - invited, under review).
- M. Di Renzo et al. "Optimization of RIS-aided MIMO – A mutually coupled loaded wire dipole model", IEEE WCL 2023 (submitted, under review).
- …

MIMO-RIS in the Presence of Scattering Objects

$$\mathbf{H}_{\text{E2E}} = \mathbf{Z}_{\text{RL}} \left[\mathbf{Z}_{\text{ROT}} - \mathbf{Z}_{\text{ROS}} \mathbf{Z}_{\text{sca}} \mathbf{Z}_{\text{SOT}} \right] \mathbf{Z}_{\text{TG}}$$

$$\mathbf{Z}_{\text{sca}} = \left(\mathbf{Z}_{\text{SS}} + \mathbf{Z}_{\text{SOS}} + \mathbf{Z}_{\text{RIS}} \right)^{-1}$$

$$\begin{aligned}
(\mathbf{P0}) \quad & \max_{\mathbf{Q}, \mathbf{Z}_{\text{RIS}}} \ \log_2 \left[\det \left(\mathbf{I}_L + \frac{\mathbf{H}_{\text{E2E}} \mathbf{Q} \mathbf{H}_{\text{E2E}}^H}{\sigma^2} \right) \right] \\
\text{s.t.} \quad & \text{Re}\{\mathbf{Z}_{\text{RIS}}(k,k)\} = R_{0,k} \geq 0, \ \ \forall k \\
& \text{Im}\{\mathbf{Z}_{\text{RIS}}(k,k)\} \in \mathcal{P}, \ \ \forall k \\
& \text{tr}(\mathbf{Q}) \leq P_t \\
& \mathbf{Q} \succcurlyeq \mathbf{0}
\end{aligned}$$

MIMO-RIS in the Presence of Scattering Objects

By keeping $\mathbf{Z}_{\text{RIS}}$ fixed, $(\mathbf{P0})$ boils down to a conventional MIMO optimization problem. Specifically, let $\mathbf{H}_{\text{E2E}} = \mathbf{U}_{\mathbf{H}_{\text{E2E}}} \boldsymbol{\Sigma}_{\mathbf{H}_{\text{E2E}}} \mathbf{V}_{\mathbf{H}_{\text{E2E}}}^{H}$ be the singular value decomposition of $\mathbf{H}_{\text{E2E}}$, where $\mathbf{V}_{\mathbf{H}_{\text{E2E}}} \in \mathbb{C}^{M \times D}$, $\mathbf{U}_{\mathbf{H}_{\text{E2E}}} \in \mathbb{C}^{L \times D}$, and $D = \text{rank}(\mathbf{H}_{\text{E2E}}) \leq \min(L, M)$. Then, the optimal $\mathbf{Q}^{\star}$ is

$$\mathbf{Q}^{\star} = \mathbf{V}_{\mathbf{H}_{\text{E2E}}} \text{diag}(p_1^{\star}, \ldots, p_D^{\star}) \mathbf{V}_{\mathbf{H}_{\text{E2E}}}^{H}$$

where $p_i^{\star} = \max\left(\left(1/\alpha - \sigma^2 / \boldsymbol{\Sigma}_{\mathbf{H}_{\text{E2E}}}(i,i)^2\right), 0\right)$, with α satisfying $\sum_{i=1}^{D} p_i^{\star} = P_t$ (water-filling power allocation).

MIMO-RIS in the Presence of Scattering Objects

By keeping $\mathbf{Q}$ fixed, the resulting optimization problem with respect to $\mathbf{Z}_{\mathrm{RIS}}$ simplifies to ($k \in \{1, \ldots, N_{\mathrm{RIS}}\}$)

$$
\begin{aligned}
(\mathbf{P1}) \quad & \max_{\mathbf{Z}_{\mathrm{RIS}}} \; \log_2 \left[\det \left(\mathbf{I}_L + \frac{\mathbf{H}_{\mathrm{E2E}} \mathbf{Q} \mathbf{H}_{\mathrm{E2E}}^H}{\sigma^2} \right) \right] \\
\text{s.t.} \quad & \mathrm{Re}\{\mathbf{Z}_{\mathrm{RIS}}(k,k)\} = R_{0,k} \geq 0, \;\; \forall k \\
& \mathrm{Im}\{\mathbf{Z}_{\mathrm{RIS}}(k,k)\} \in \mathcal{P}, \;\; \forall k
\end{aligned}
$$

MIMO-RIS in the Presence of Scattering Objects

By keeping $\mathbf{Q}$ fixed, the resulting optimization problem with respect to $\mathbf{Z}_{\text{RIS}}$ simplifies to ($k \in \{1, \ldots, N_{\text{RIS}}\}$)

$$
\begin{aligned}
(\mathbf{P1}) \quad & \max_{\mathbf{Z}_{\text{RIS}}} \ \log_2 \left[\det \left(\mathbf{I}_L + \frac{\mathbf{H}_{\text{E2E}} \mathbf{Q} \mathbf{H}_{\text{E2E}}^H}{\sigma^2} \right) \right] \\
\text{s.t.} \quad & \operatorname{Re}\{\mathbf{Z}_{\text{RIS}}(k,k)\} = R_{0,k} \geq 0, \quad \forall k \\
& \operatorname{Im}\{\mathbf{Z}_{\text{RIS}}(k,k)\} \in \mathcal{P}, \quad \forall k
\end{aligned}
$$

- **The approach consists of three steps:**
  - **Sherman-Morrison's formula**
  - **Sylvester's determinant theorem**
  - **Gram-Schmidt's orthogonalization process**

MIMO-RIS in the Presence of Scattering Objects

$$\mathbf{Z}_{\mathrm{sca}} = \left(\mathbf{Z}_{\mathrm{SS}} + \mathbf{Z}_{\mathrm{SOS}} + \mathbf{Z}_{\mathrm{RIS}}\right)^{-1}$$

MIMO-RIS in the Presence of Scattering Objects

$$\mathbf{Z}_{\mathrm{sca}} = \left(\mathbf{Z}_{\mathrm{SS}} + \mathbf{Z}_{\mathrm{SOS}} + \mathbf{Z}_{\mathrm{RIS}}\right)^{-1}$$

$$\mathbf{Z}_{\mathrm{sca}} = \left(\mathbf{Z}_{\mathrm{SS}} + \mathbf{Z}_{\mathrm{SOS}} + \mathbf{Z}_{\mathrm{RIS},k} + \mathbf{Z}_{\mathrm{RIS}}(k,k)\mathbf{e}_k\mathbf{e}_k^T\right)^{-1}$$

MIMO-RIS in the Presence of Scattering Objects

$$\mathbf{Z}_{\text{sca}} = \left(\mathbf{Z}_{\text{SS}} + \mathbf{Z}_{\text{SOS}} + \mathbf{Z}_{\text{RIS}}\right)^{-1}$$

$$\mathbf{Z}_{\text{sca}} = \left(\mathbf{Z}_{\text{SS}} + \mathbf{Z}_{\text{SOS}} + \mathbf{Z}_{\text{RIS},k} + \mathbf{Z}_{\text{RIS}}(k,k)\mathbf{e}_k\mathbf{e}_k^T\right)^{-1}$$

$$\mathbf{A}_k = \mathbf{Z}_{\text{SS}} + \mathbf{Z}_{\text{SOS}} + \mathbf{Z}_{\text{RIS},k}, \quad z_k = \mathbf{Z}_{\text{RIS}}(k,k)$$

MIMO-RIS in the Presence of Scattering Objects

- **The approach consists of three steps:**
  - **Sherman-Morrison's formula**
  - **Sylvester's determinant theorem**
  - **Gram-Schmidt's orthogonalization process**

$$\mathbf{Z}_{\text{sca}} = \mathbf{Z}_{\text{sca}}\left(z_k\right) = \mathbf{A}_k^{-1} - \frac{\mathbf{A}_k^{-1}\mathbf{e}_k\mathbf{e}_k^T\mathbf{A}_k^{-1}}{1 + z_k\mathbf{e}_k^T\mathbf{A}_k^{-1}\mathbf{e}_k} z_k$$

MIMO-RIS in the Presence of Scattering Objects

- The approach consists of three steps:
  - Sherman-Morrison's formula
  - Sylvester's determinant theorem
  - Gram-Schmidt's orthogonalization process

$$R(z_k) = \log_2 \left[\det \left(\mathbf{I}_L + \frac{\mathbf{B}_k \mathbf{Q} \mathbf{B}_k^H}{\sigma^2} \right) \right] + \log_2 \left[\det (\mathbf{S}_k (z_k)) \right]$$

$$\mathbf{S}_k (z_k) = \mathbf{I}_L + \mathbf{U}_k \mathbf{\Sigma}_k^{-1} \mathbf{U}_k^H (\mathbf{X}_1 (z_k) + \mathbf{X}_2 (z_k))$$

$$\det(\mathbf{S}_k (z_k)) = \det \left(\mathbf{I}_L + \mathbf{\Sigma}_k^{-\frac{1}{2}} \mathbf{U}_k^H (\mathbf{X}_1 (z_k) + \mathbf{X}_2 (z_k)) \mathbf{U}_k \mathbf{\Sigma}_k^{-\frac{1}{2}} \right)$$

MIMO-RIS in the Presence of Scattering Objects

- **The approach consists of three steps:**
  - **Sherman-Morrison's formula**
  - **Sylvester's determinant theorem**
  - **Gram-Schmidt's orthogonalization process**

$$\mathbf{S}_k(z_k) = \mathbf{I}_L + \frac{\tilde{\mathbf{u}}_k \tilde{\mathbf{v}}_k^H}{\sigma^2 \chi_k(z_k)} + \frac{\tilde{\mathbf{v}}_k \tilde{\mathbf{u}}_k^H}{\sigma^2 \chi_k^*(z_k)} + \frac{\tilde{\mathbf{u}}_k \tilde{\mathbf{u}}_k^H \mathbf{v}_k^H \mathbf{Q} \mathbf{v}_k}{\sigma^2 |\chi_k(z_k)|^2}$$

$$\tilde{\mathbf{u}}_k = \mathbf{\Sigma}_k^{-\frac{1}{2}} \mathbf{U}_k^H \mathbf{u}_k, \quad \tilde{\mathbf{v}}_k^H = \mathbf{v}_k^H \mathbf{Q} \mathbf{B}_k^H \mathbf{U}_k \mathbf{\Sigma}_k^{-\frac{1}{2}}$$

The vectors u_k and v_k are not orthogonal → orthogonalization

$$\mathbf{t}_1 = \frac{1}{||\tilde{\mathbf{u}}_k||} \tilde{\mathbf{u}}_k, \quad \mathbf{t}_2 = \frac{1}{||\mathbf{t}||} \mathbf{t}$$

$$\mathbf{t} = \tilde{\mathbf{v}}_k - \frac{\langle \tilde{\mathbf{v}}_k, \mathbf{t}_1 \rangle}{\langle \mathbf{t}_1, \mathbf{t}_1 \rangle} \mathbf{t}_1 = \frac{||\tilde{\mathbf{u}}_k||^2 \tilde{\mathbf{v}}_k - \tilde{\mathbf{u}}_k^H \tilde{\mathbf{v}}_k \tilde{\mathbf{u}}_k}{||\tilde{\mathbf{u}}_k||^2}$$

MIMO-RIS in the Presence of Scattering Objects

- **The approach consists of three steps:**
  - **Sherman-Morrison's formula**
  - **Sylvester's determinant theorem**
  - **Gram-Schmidt's orthogonalization process**

$$\det(\mathbf{S}_k) = \det\left(\begin{pmatrix} \mathbf{t}_1 & \mathbf{t}_2 \end{pmatrix} \begin{pmatrix} 1 + s_k(z_k) & \frac{||\tilde{\mathbf{u}}_k|| \tilde{\mathbf{v}}_k^H \mathbf{t}_2}{\sigma^2 \chi_k(z_k)} \\ \frac{||\tilde{\mathbf{u}}_k|| \mathbf{t}_2^H \tilde{\mathbf{v}}_k}{\sigma^2 \chi_k^*(z_k)} & 1 \end{pmatrix} \begin{pmatrix} \mathbf{t}_1^H \\ \mathbf{t}_2^H \end{pmatrix} \right)$$

$$= \det \begin{pmatrix} 1 + s_k(z_k) & \frac{||\tilde{\mathbf{u}}_k|| \tilde{\mathbf{v}}_k^H \mathbf{t}_2}{\sigma^2 \chi_k(z_k)} \\ \frac{||\tilde{\mathbf{u}}_k|| \mathbf{t}_2^H \tilde{\mathbf{v}}_k}{\sigma^2 \chi_k^*(z_k)} & 1 \end{pmatrix} = \det(\mathbf{W})$$

$$s_k(z_k) = \frac{||\tilde{\mathbf{u}}_k|| \tilde{\mathbf{v}}_k^H \mathbf{t}_1}{\sigma^2 \chi_k(z_k)} + \frac{||\tilde{\mathbf{u}}_k|| \mathbf{t}_1^H \tilde{\mathbf{v}}_k}{\sigma^2 \chi_k^*(z_k)} + \frac{||\tilde{\mathbf{u}}_k||^2 \mathbf{v}_k^H \mathbf{Q} \mathbf{v}_k}{\sigma^2 |\chi_k(z_k)|^2}$$

MIMO-RIS in the Presence of Scattering Objects

Since $\mathbf{W} = \mathbf{W}(z_k)$ is a 2×2 matrix, the determinant of $\mathbf{S}_k = \mathbf{S}_k(z_k)$ can be expressed in closed-form as

$$\det(\mathbf{S}_k(z_k)) = 1 + \frac{c_1}{\chi_k(z_k)} + \frac{c_1^*}{\chi_k^*(z_k)} + \frac{c_2}{|\chi_k(z_k)|^2}$$

where

$$c_1 = \frac{||\tilde{\mathbf{u}}_k||\tilde{\mathbf{v}}_k^H \mathbf{t}_1}{\sigma^2}, \; c_2 = \frac{||\tilde{\mathbf{u}}_k||^2 \mathbf{v}_k^H \mathbf{Q} \mathbf{v}_k}{\sigma^2} - \frac{||\tilde{\mathbf{u}}_k||^2 |\tilde{\mathbf{v}}_k^H \mathbf{t}_2|^2}{\sigma^4}$$

In conclusion, since $\chi_k(z_k) = 1 + a_k z_k$, with $z_k = R_{0,k} + jX_k$ and $R_{0,k}$ is assumed known and fixed, $(\mathbf{P1})$ boils down to maximizing the single-variable (i.e., X_k) function

$$f(X_k) = 1 + \frac{c_1}{1 + a_k(R_{0,k} + jX_k)} + \frac{c_1^*}{1 + a_k^*(R_{0,k} + jX_k)^*} + \frac{c_2}{|1 + a_k(R_{0,k} + jX_k)|^2}, \quad X_k \in \mathcal{P}$$

MIMO-RIS in the Presence of Scattering Objects

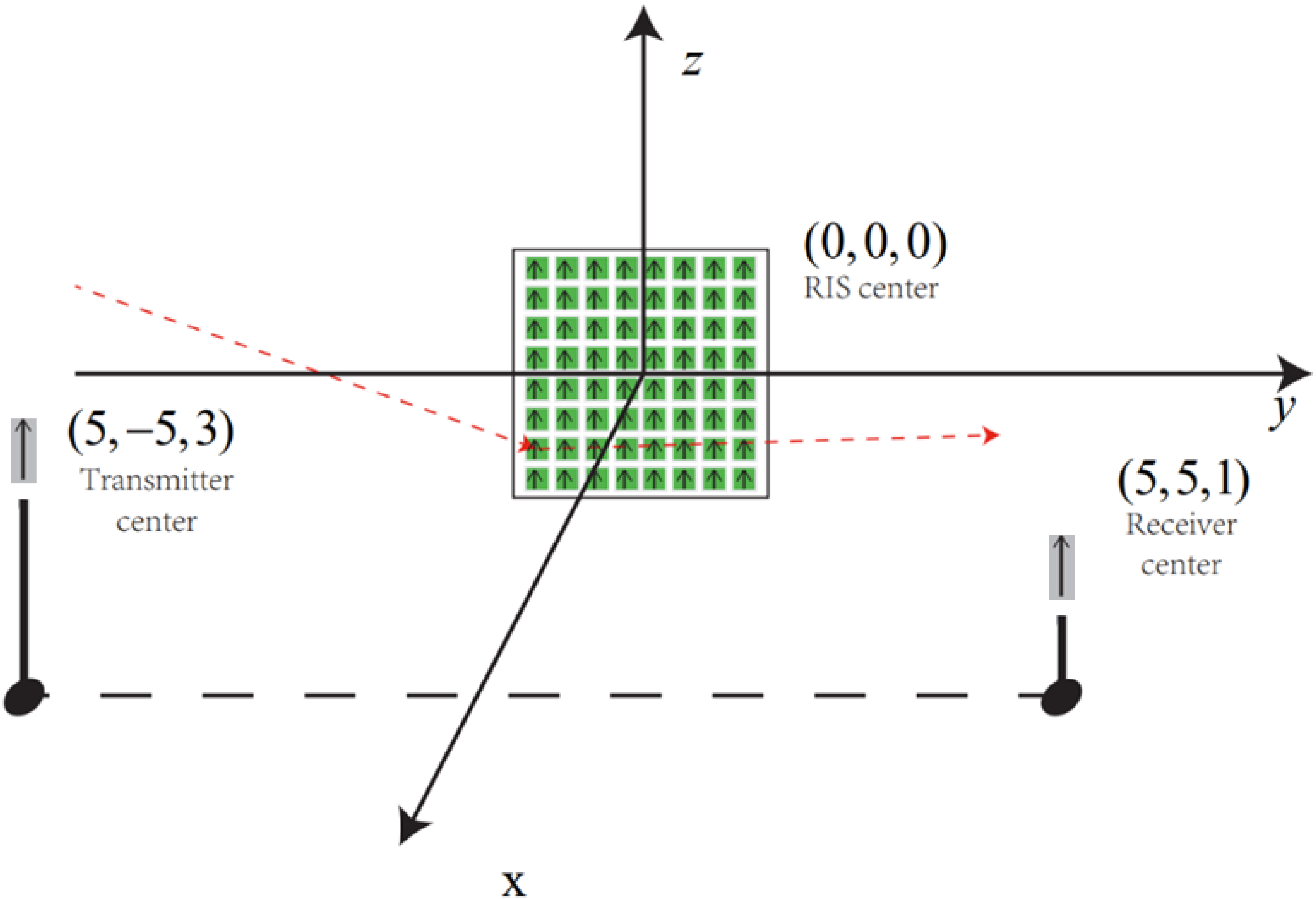
Algorithm 1 Proposed algorithm for solving $(\mathbf{P0})$

Input: Compute the impedance matrices from [4, Lemma 2];
Initialize: $q = 0$, $\epsilon \geq 0$, $\mathbf{r}_0 = [R_{0,1}, \ldots, R_{0,N_{\text{RIS}}}]^T$, $\mathbf{x}^{(0)} = [X_1^{(0)}, \ldots, X_{N_{\text{RIS}}}^{(0)}]^T \in \mathcal{P}^{N_{\text{RIS}}}$, $R^{(-1)} = 0$, $R^{(0)} = R(\mathbf{Q}^{(0)}, \mathbf{Z}^{(0)})$ with $\mathbf{Z}^{(0)} = \text{diag}(\mathbf{r}_0) + j\,\text{diag}(\mathbf{x}^{(0)})$ and $R(\cdot,\cdot)$ defined in (8);
while $|R^{(q)} - R^{(q-1)}| > \epsilon$ **do**
  Compute $\mathbf{Q}^\star$ from (14);
  for $k = 1, \ldots, N_{\text{RIS}}$ **do**
    Compute $X_k^\star$ from Proposition 1;
    Update $\mathbf{Z}_{\text{RIS}}^\star(k,k) \leftarrow R_{0,k} + jX_k^\star$;
  end for
  $q = q + 1$, $R^{(q)} = R(\mathbf{Q}^\star, \mathbf{Z}_{\text{RIS}}^\star)$;
end while
Return: $\mathbf{Q}^\star$ and $\mathbf{Z}_{\text{RIS}}^\star$.

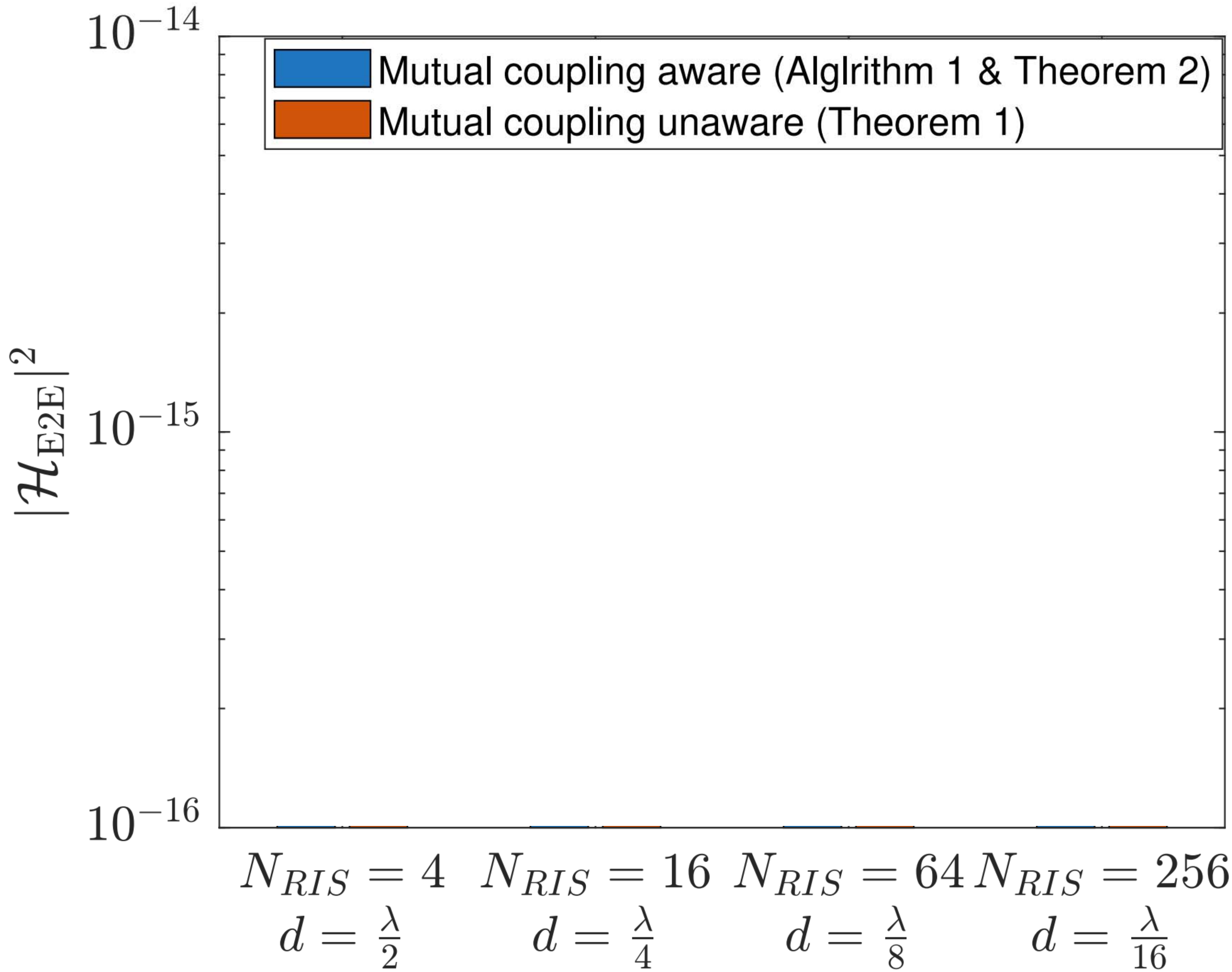
Numerical Examples

Optimization in Free Space

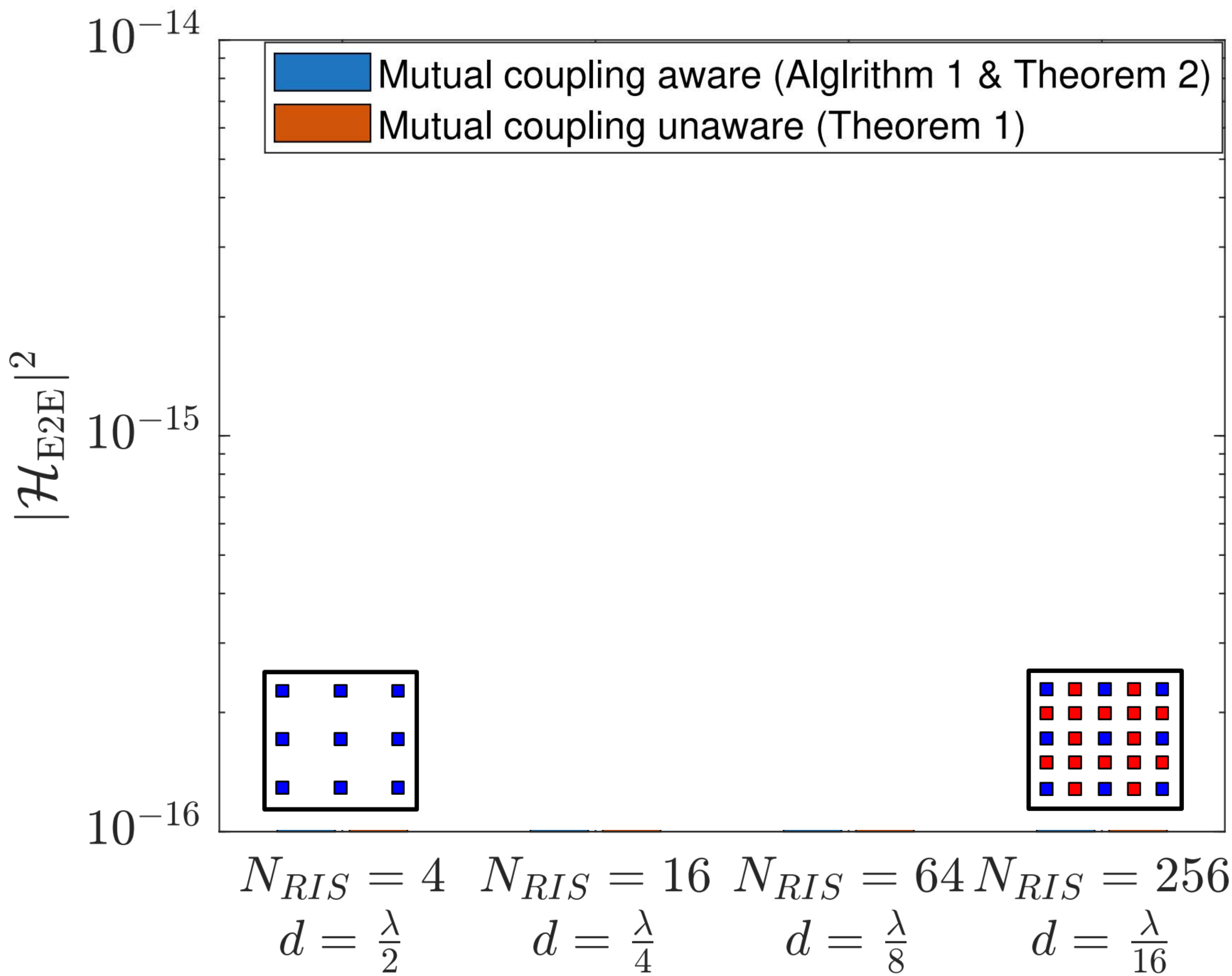
Single-User SISO



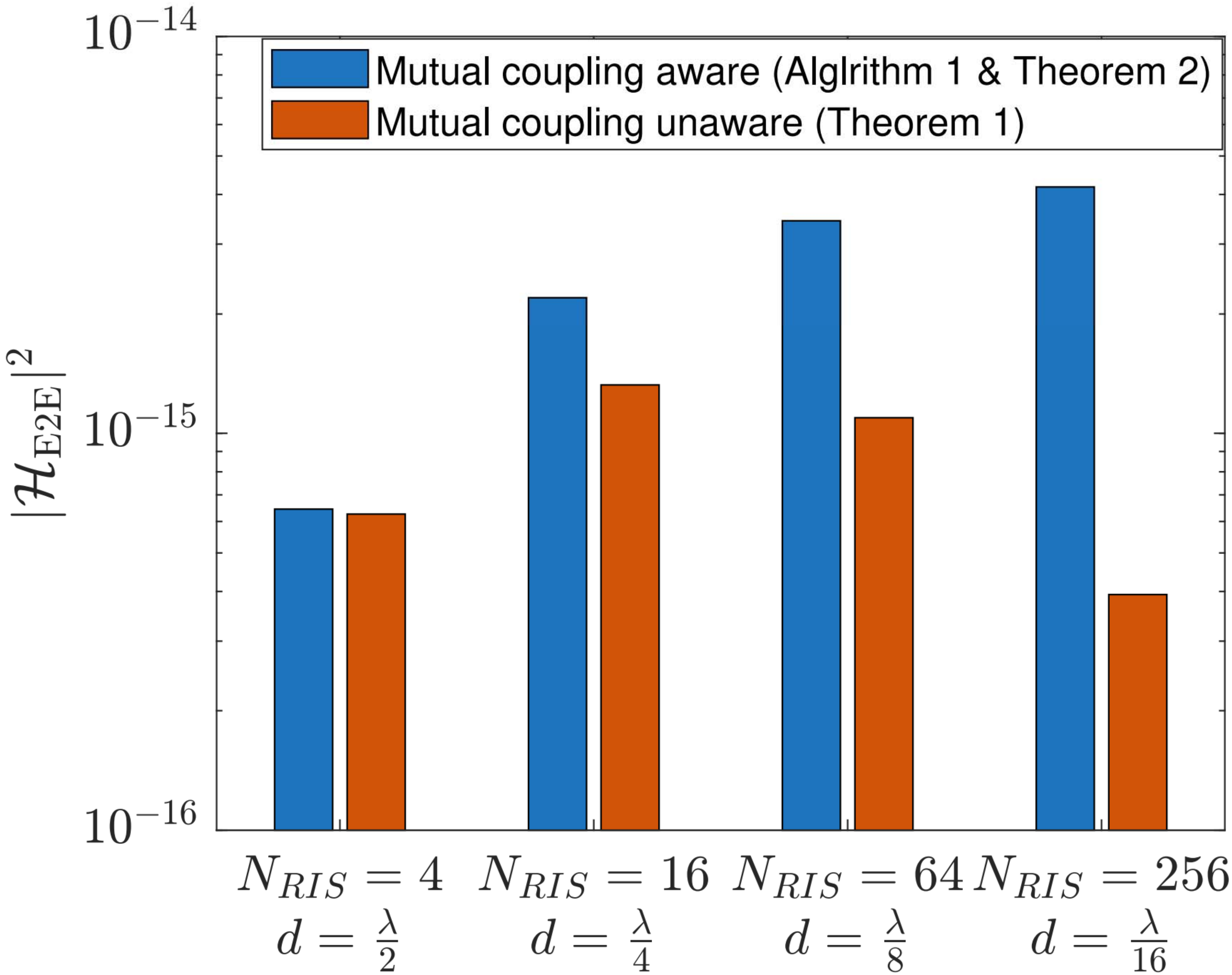
Optimization in Free Space (SISO, Fixed Size RIS)



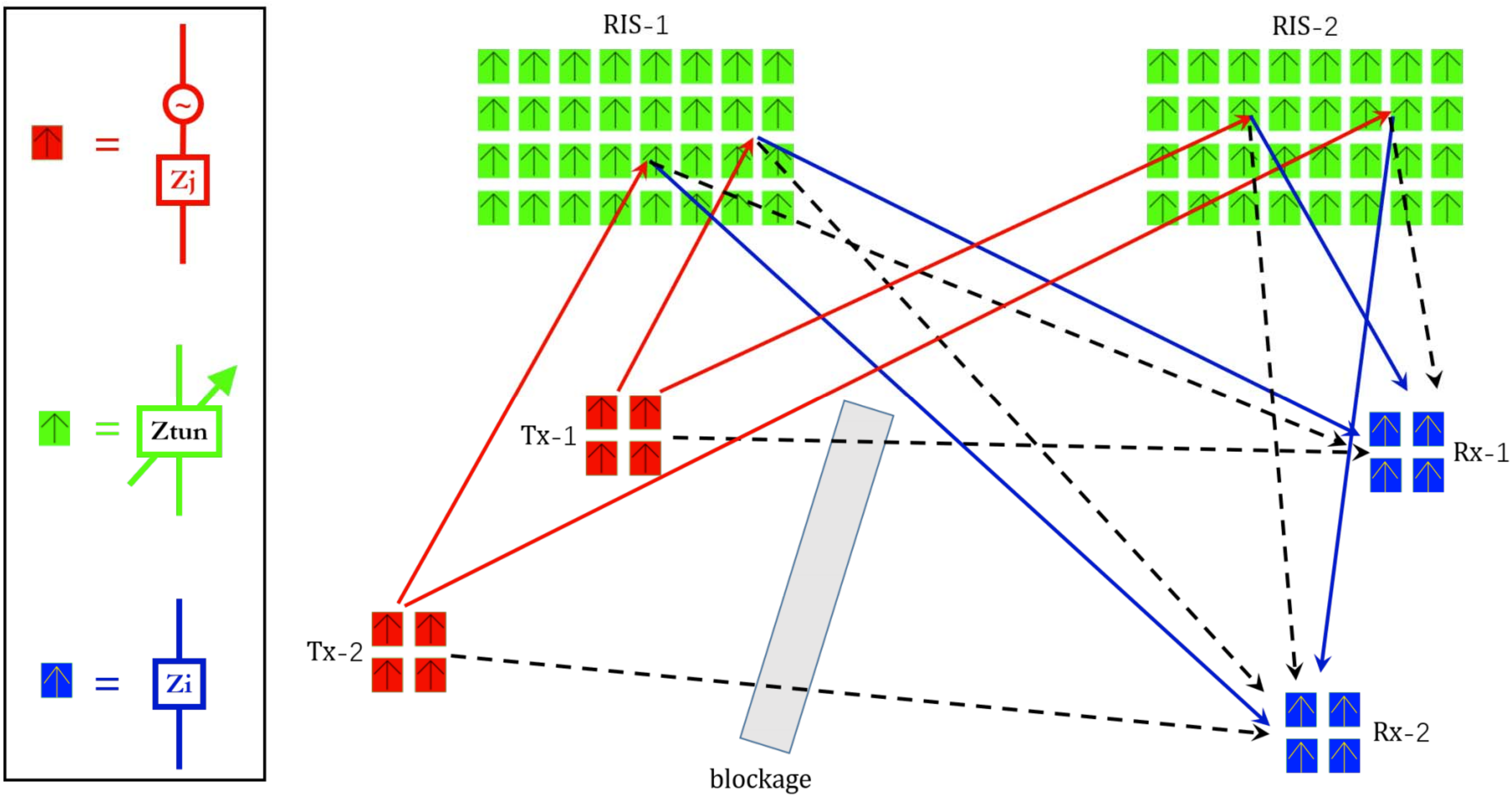
Optimization in Free Space (SISO, Fixed Size RIS)



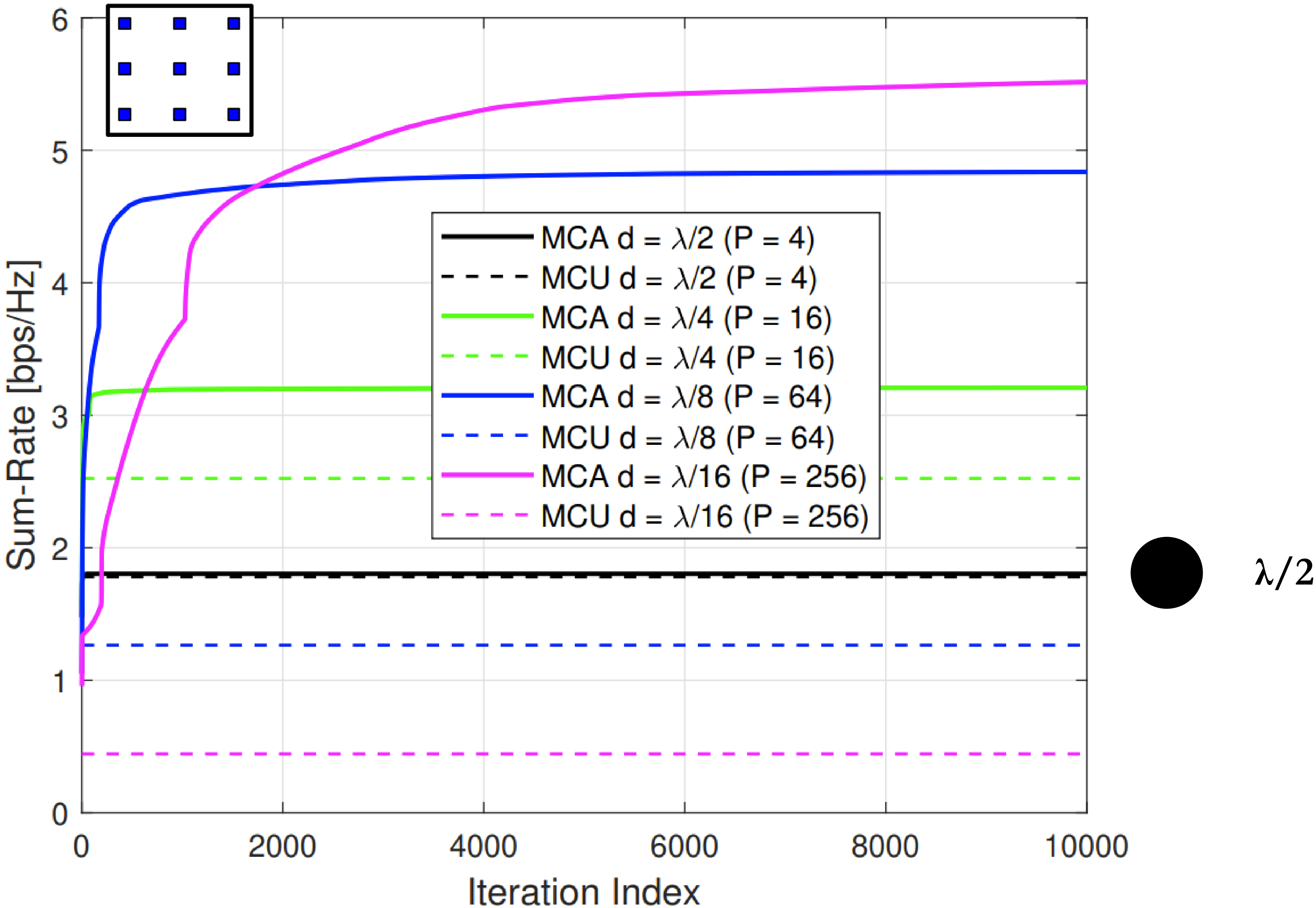
Optimization in Free Space (SISO, Fixed Size RIS)



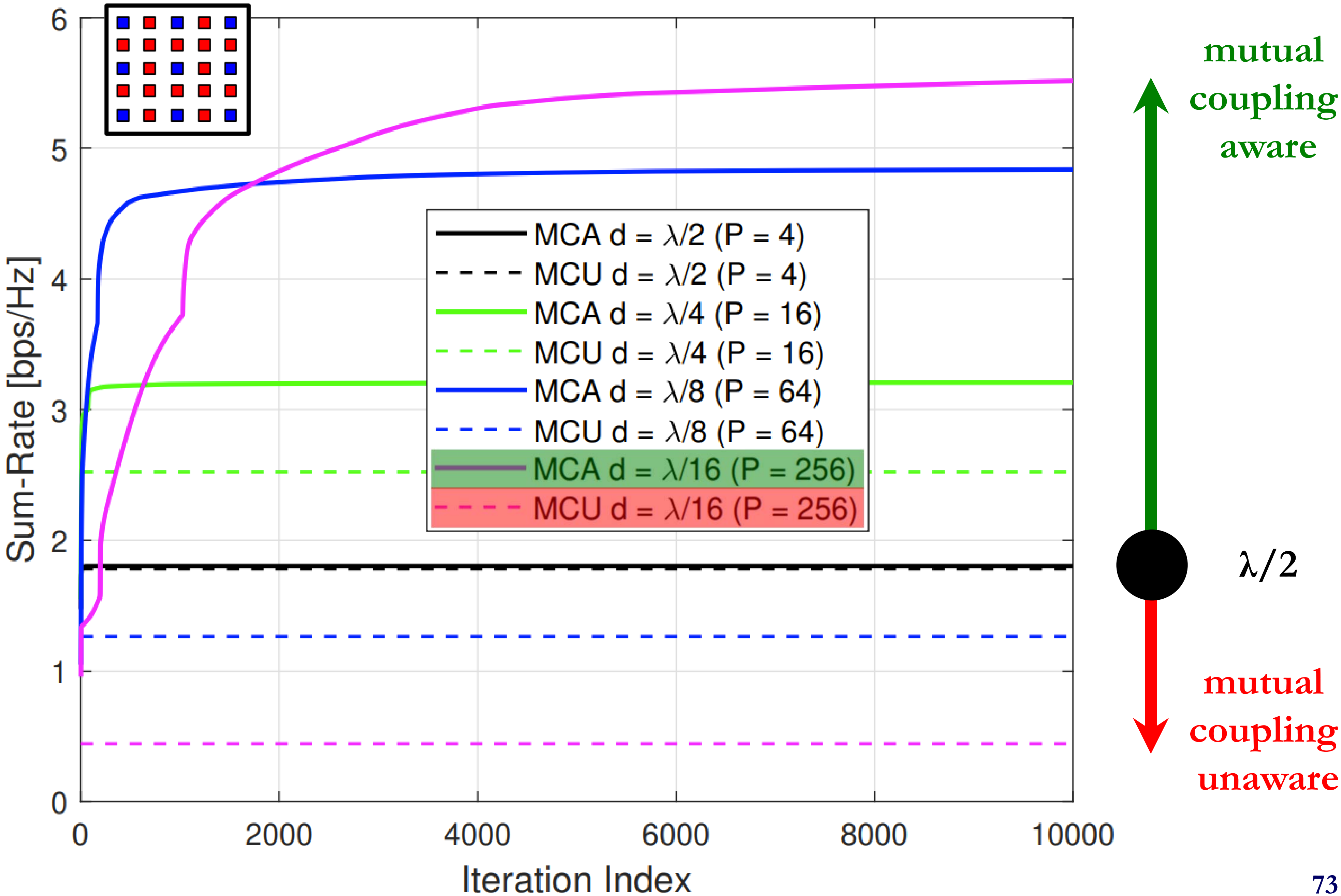
Multi-User MIMO



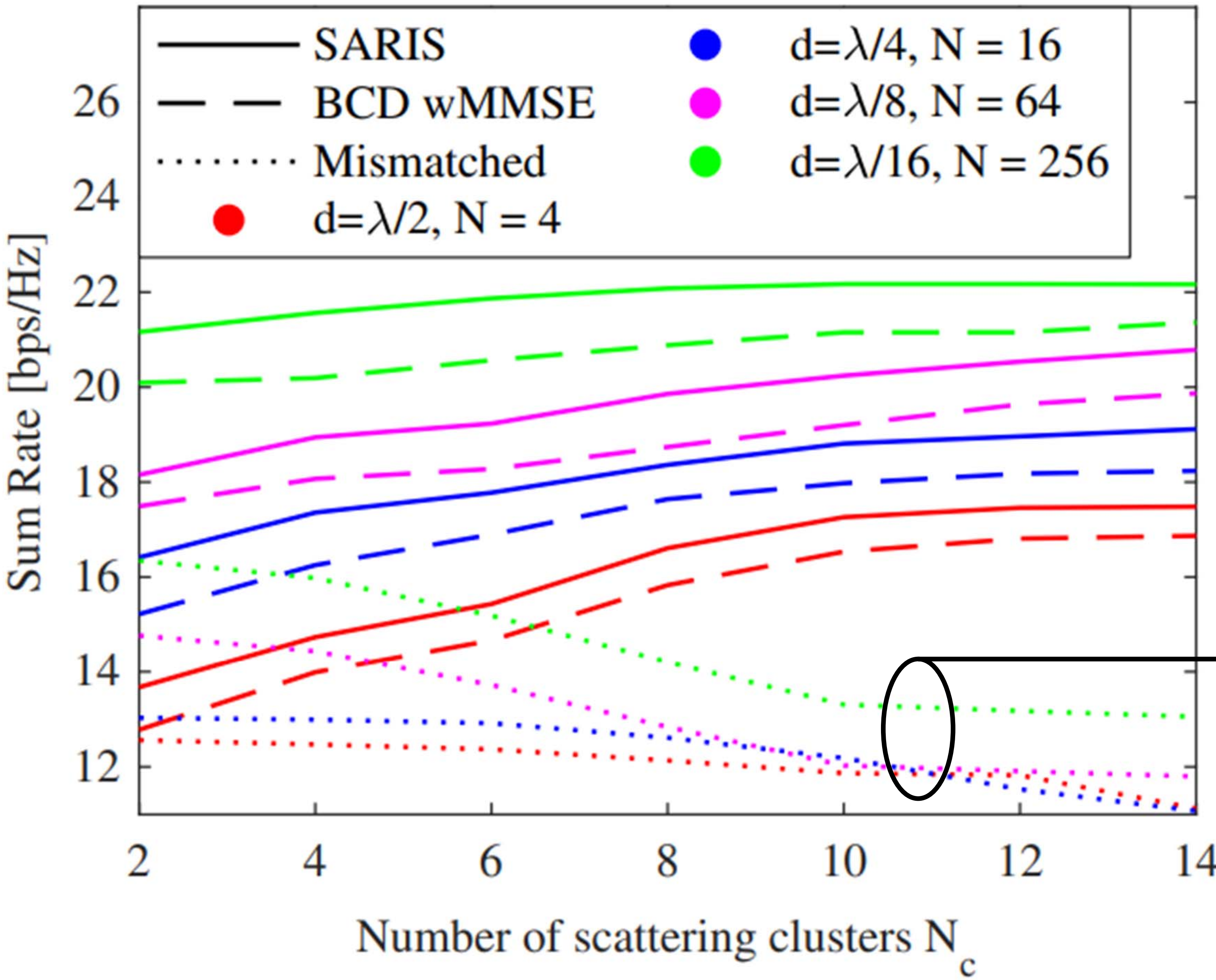
Optimization in Free Space (MIMO, Fixed Size RIS)



Optimization in Free Space (MIMO, Fixed Size RIS)

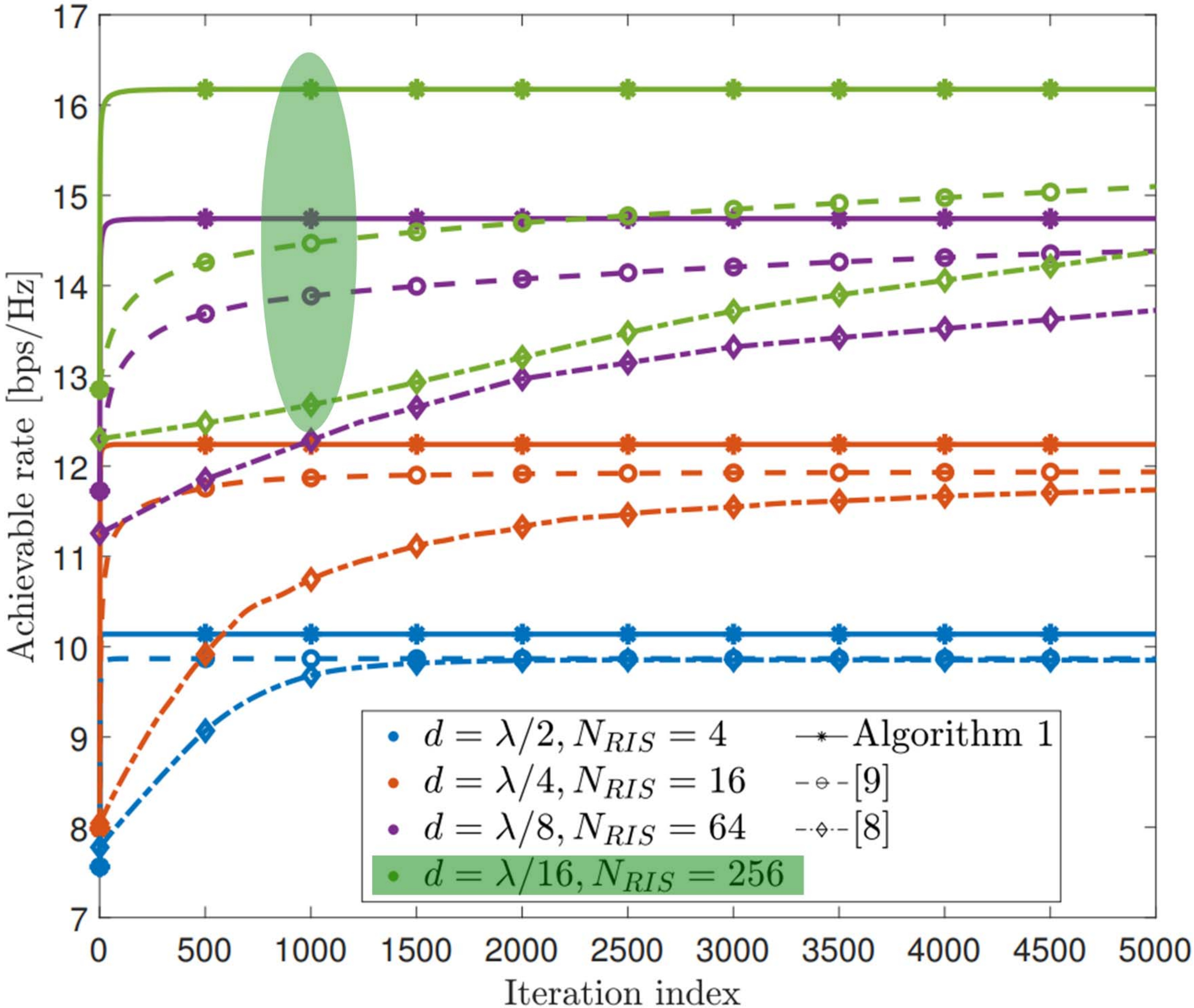


MIMO-RIS Optimization with Scattering Objects



ignoring the interaction between the RIS and the objects at the design stage

MIMO-RIS Optimization with Scattering Objects



MIMO-RIS Optimization with Scattering Objects

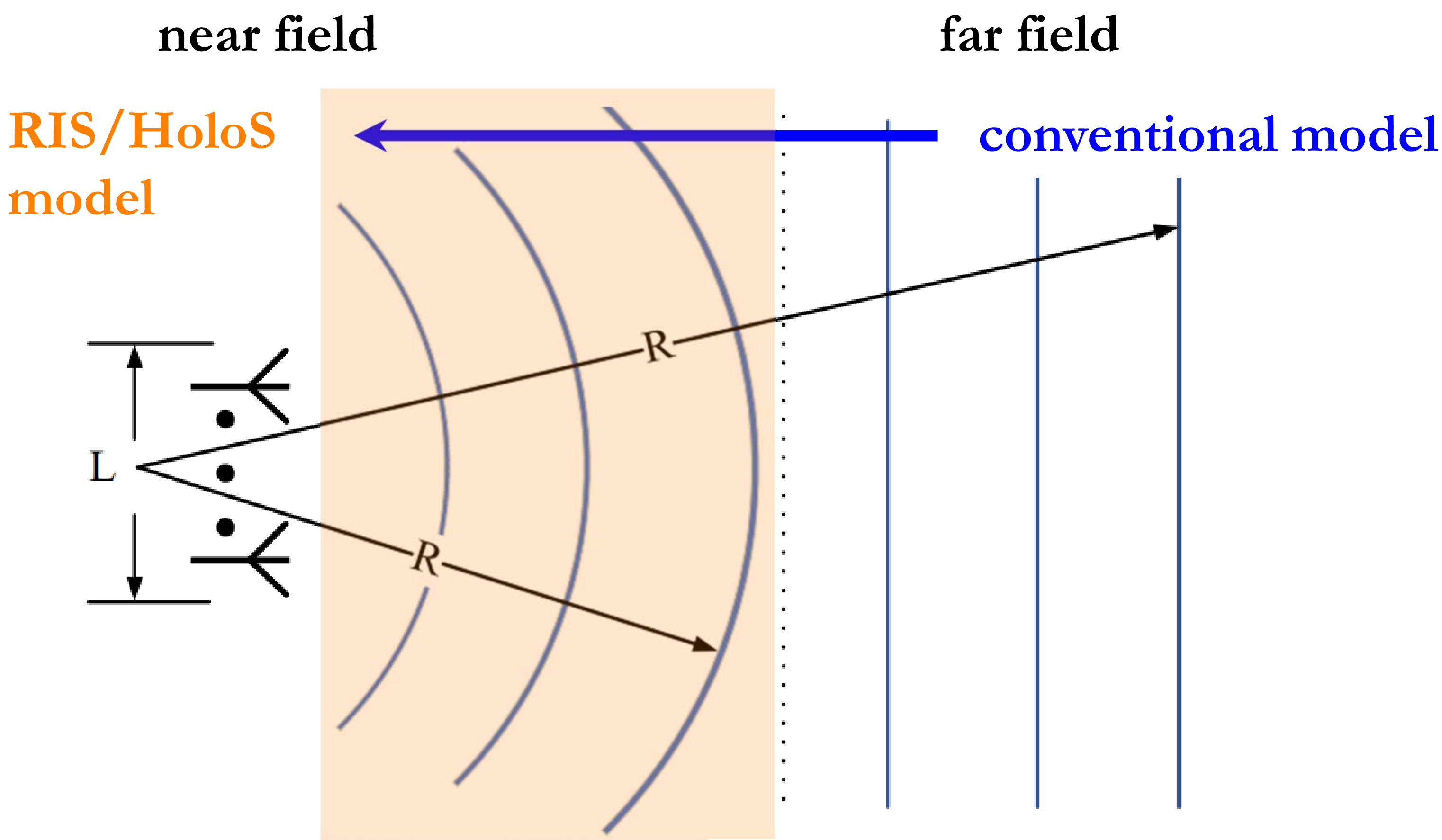
Table II: Comparison of the execution time [seconds]

d	**Algo. 1** (90%)	**[8]**
$\lambda/2$	0.001	0.800
$\lambda/4$	0.004	0.770
$\lambda/8$	0.167	8.135
$\lambda/16$	16.530	213.128

d	**Algo. 1** (98%)	**[9]**
$\lambda/2$	0.001	0.0154
$\lambda/4$	0.008	0.896
$\lambda/8$	0.834	27.686
$\lambda/16$	170.807	946.404

Conclusion: Approach to Model the Near Field

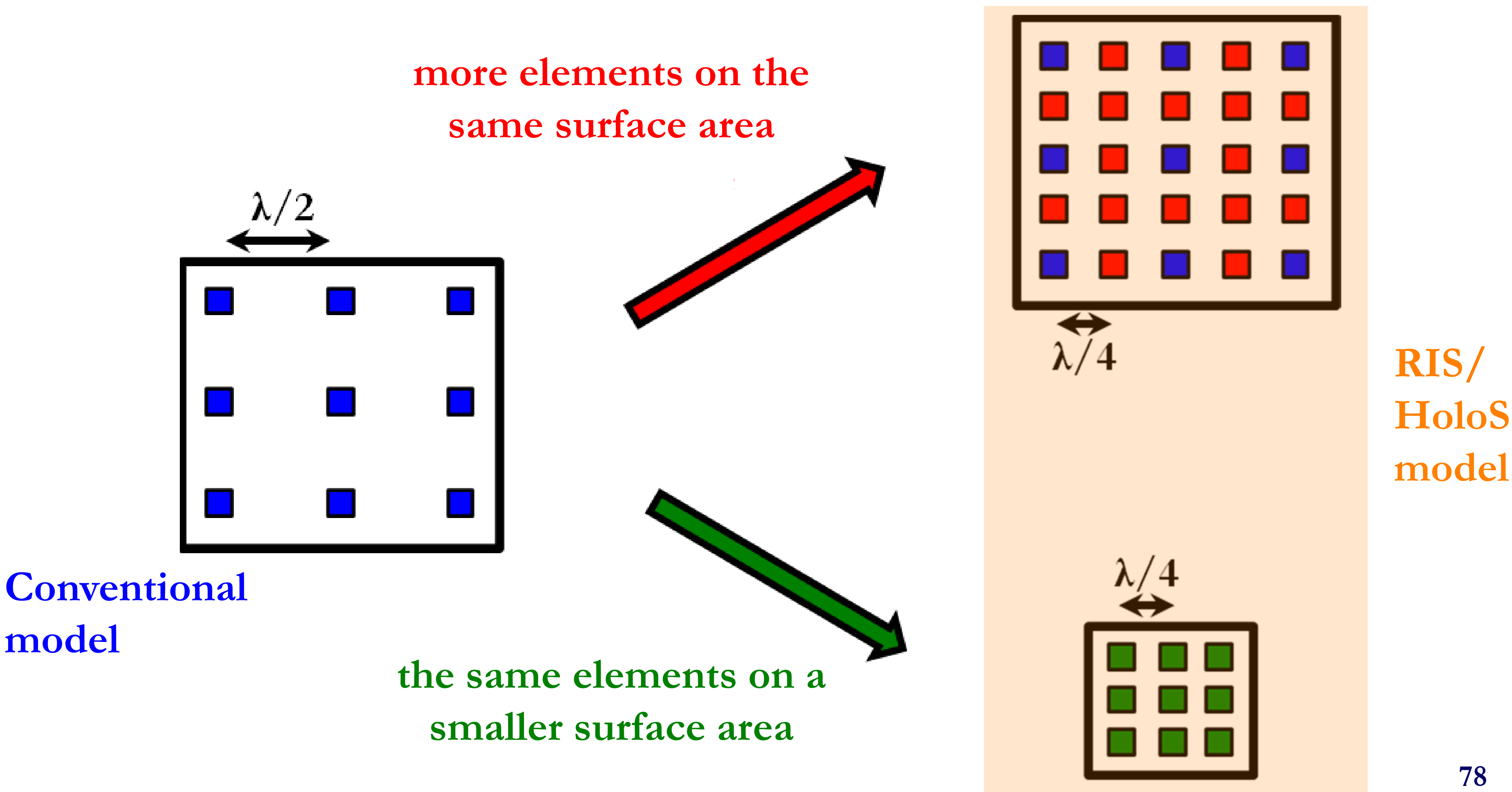
- **Paradigm #1: The wavefronts of the electromagnetic waves are (approximated as) locally planar on the antenna arrays**
  - **RISs/HoloS are electrically large and the transmission distances are shrinking**



$$R \geq R_f = \frac{2L^2}{\lambda}$$

Conclusion: Approach to Model the Mutual Coupling

- **Paradigm #2: The radiating elements of antenna-arrays are decoupled electromagnetically**
  - The inter-distances are smaller than the wavelength (< λ/2)



Programmable Metasurfaces for Wireless Communications: A Loaded Thin Wire Model

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